APPENDIX F

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APPENDIX F

F-000 Improvement Curve Analysis Techniques

F-001 Scope of Appendix

As explained later in this appendix, the improvement (or learning) curve is a generalization of the concept that, within certain reasonable limits, the knowledge, skills, and techniques employed in the production of a product will improve as production of the product continues without material change and that this improvement will result in corresponding reduction in the time and material required to produce the product and, therefore, in the cost of the product. The generalization also postulates that the rate of improvement will be relatively regular and constant for any given product. By stating these concepts as generalizations, a valuable technique of graphical and computational analysis and a tool for evaluating production requirements and costs has been made available to production planners, analysts, and contract auditors. This appendix discusses the methods of using these techniques in the evaluation of contract production costs. The principles underlying these techniques and their use in analyzing costs are extensions of the principles discussed in Appendix E.

F-100 Section 1 --- The Improvement Curve Theory

F-101 Introduction

This section discusses the improvement curve theory including its concept, description, and characteristics.

F-102 Concept

a. The improvement curve is a statistical device used in predicting production costs and as an aid in planning and controlling production. The theory of the curve assumes a predictable correlation between the number of manhours (or the labor, material, or other cost) necessary to produce a particular unit or a particular production lot of units and the number of such units or lots successively produced. The term is also applied to the line graph which depicts this correlation and to the computational procedures for estimating the cost or manhour requirements under the improvement curve theory. The line graph and improvement curve theory are both based on the principle that the time required to produce (and, therefore, other things being equal, the cost of producing) successive quantities of a product decreases with additional experience and the introduction of improved methods and tools. The term "improvement curve" is derived from the fact that the curve reflects this decrease. Because the reduction is largely a result of increased knowledge and skill, the curve and its theory are sometimes referred to as the learning curve, the experience curve, or the progress curve.

b. The principles of a gradual reduction in the unit cost of a product as production continues has long been accepted but the use of the improvement curve as a management and auditing tool for evaluating and forecasting costs, and for planning and controlling production, is an outgrowth of research in the fields of defense procurement and production conducted primarily by the airframe industry and the Government. The correlation of production quantities and direct labor requirements for the production of airplanes disclosed for the firms studied that as production of a particular product continued there was a relatively constant percentage reduction in the labor requirements for doubled quantities of production. For example, a study of production costs incurred by various airframe contractors during World War II revealed that the average rate of improvement (that is, the average rate of reduction in labor requirements) for all companies studied was 20% between successive doubled quantities. In other words, the labor required to build the second plane was approximately 80% of that required to build the first; the labor required for the fourth plane was approximately 80% of that to build the second; the labor for the eighth, approximately 80% of that for the fourth; and so on for each successive doubling of production. During this same period, the calculated rate of improvement for individual companies in the airframe industry varied from 2% to approximately 35%. Expressed in another manner, the cost of successive doubled quantities of production varied from 98% to 65% of the cost of the preceding quantities. The improvement curve theory as presently used by the industry and the Government assumes this basic relationship: that there will be a relatively constant percentage reduction in the cost for doubled quantities of production. "Cost" as used in this paragraph and in all subsequent references to the cost-quantity relationship refers to a cost that may be expressed in terms of dollars or where appropriate, in terms of a quantity such as labor-hours. This latter method of expressing the cost of a product, especially of direct labor, is frequently preferred because it eliminates the effect of extraneous factors such as changes in labor rates.

c. The original studies and applications of the improvement curve were confined to the direct labor requirements for building airplanes, but subsequent experience indicates that there are similar patterns of improvement for other production costs (such as material costs) and in other industries, especially where hand or line operations are involved. Thus, today the improvement curve theory may be applied in the audit evaluation of costs and cost estimates in any industry, provided that the basic assumption of a relatively constant rate of improvement can be shown to be true for the particular cost-quantity relationships being studied. When this relationship is valid for any element of the cost of producing an item, the improvement curve pattern experienced in the production of the item in the past can be extended to obtain predictions of the costs which will be required to produce additional units in the future. A further assumption relative to these elements may sometimes be made; namely, that the rate of improvement experienced by a particular contractor on a prior product may be indicative of the rate of improvement which can be expected on a new product of similar size, complexity, and construction. When both of these assumptions are valid, the use of the improvement curve simplifies the problem of evaluating an estimated cost for a new product and permits a more sound evaluation than is possible without the use of the curve. Without the improvement curve technique, the auditor must attempt to evaluate directly either the total cost or the overall average cost for the entire future production. This direct evaluation of an estimate is difficult if the estimate covers an extended period of time even though past cost experience is available. It is more difficult for a new product. Where the improvement curve assumptions are valid, however, the auditor can first evaluate the actual or estimated initial cost of manufacture and from this information he or she can evaluate both the expected total and the average costs for the production period by using the improvement curve theory.

d. A number of factors, such as those listed below, contribute toward a progressive increase in efficiency as production of a given product continues and thus account for a corresponding and progressive decline in unit costs. Their effect on the improvement curve is discussed more fully in F-106b.

(1) Job familiarization by both production workers and supervisory personnel.

(2) Changes in product design which do not materially affect the product, but result in increased ease and speed of production.

(3) Changes in tooling, machinery, and equipment which simplify or speed up the production process.

(4) Improved production planning and scheduling, and improvements in production techniques and operational methods.

(5) Improvements in shop organization, and in engineering coordination and liaison.

(6) Improvements in the handling and flow of materials, and in the materials and parts supply systems, with an attendant reduction in lost time.

F-103 Description of the Improvement Curve

a. Figure F-1-1 is an illustration of an idealized 80% improvement curve (a 20% rate of improvement) plotted on arithmetic graph paper and based on the man-hour data contained in the following table. For simplicity, the table is based on the assumption that the first unit required 100 person-hours to produce. It will be observed that the table indicates a constant rate of reduction of 20% for each doubling of the unit number; the value of the second and each succeeding item in the table is 80% of the value of the preceding item. The curve drawn through the plotted points in Figure F-1-1 dramatically reveals this reduction in person-hours as succeeding units are produced. At first it dips sharply because the amount of reduction per unit is large. As production continues the reduction per unit becomes smaller, and the line begins to slope downward more gently as the distance between doubled quantities becomes progressively larger. Thus, as production continues, the curve, when plotted on arithmetic paper, tends to approach the horizontal but theoretically does not actually become horizontal.

TABLE FOR FIGURES F-1-1 and F-1-2

Unit No.	Unit Person-hours
1	100.00
2	80.00
4	64.00
8	51.20
16	40.96
32	32.77
64	26.21

Figure F-1-1
80% Unit Curve on Arithmetic Paper

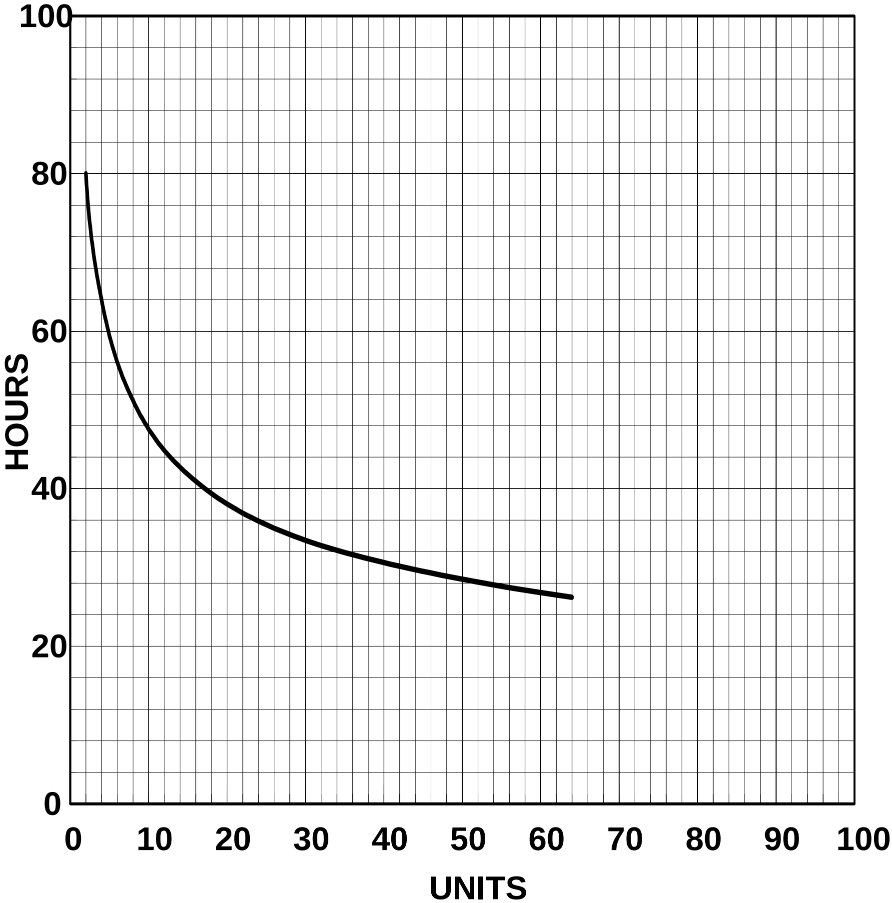
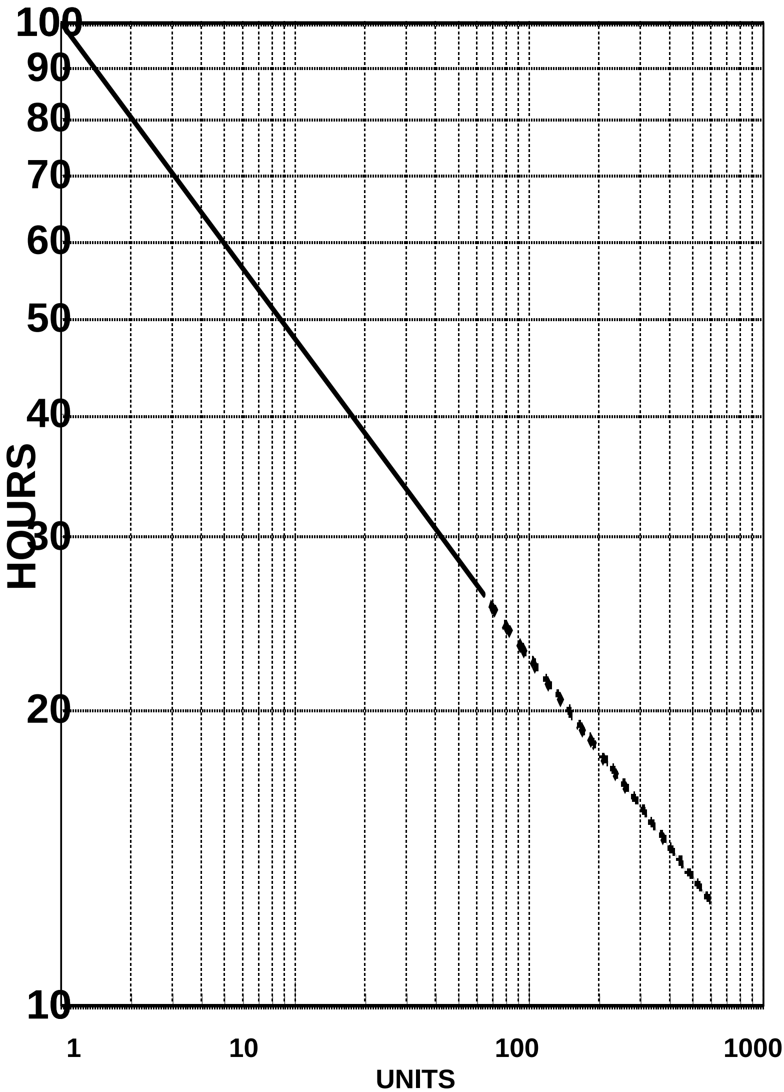


Figure F-1-2
80% Unit Curve on Logarithmic Paper



b. An improvement curve plotted on arithmetic paper has the advantage of displaying the pattern of costs incurred in normal perspective. This is particularly useful in assessing the effects of engineering changes and other descriptions. However, it is difficult to determine from a graph drawn on arithmetic paper if there is a constant reduction for doubled quantities; and it is difficult to measure the rate of reduction. Further, as each unit is represented by an equal distance, a graph showing a series of several hundred or several thousand units would be excessively long and impractical. Construction, interpretation, and projection of a curve on arithmetic paper are also difficult, especially for the portion of the curve pertaining to early production quantities where the slope of the curve changes rapidly. For these reasons the improvement curve should be plotted on full logarithmic scale (log-log) paper where, as will be shown later, the curve becomes a straight line.

c. On an arithmetic scale equal amounts are represented by equal distances (Figure F-1-1). In contrast, on a logarithmic scale the distances between doubled amounts are equal (Figure F-1-2). An improvement curve, therefore, which very closely follows the improvement curve theory will be approximately a straight line when plotted on log-log paper; a fact which facilitates interpretation and projection of the curve. Figure F-1-2 illustrates an improvement curve drawn on log-log paper. The solid portion of the curve was plotted from the data previously used in constructing Figure F-1-1. A projection of the curve through unit 500 is shown by a broken line. Improvement curves of 70%, 80%, and 90% drawn on log-log paper; are illustrated in Figure F-1-3. These curves show that the more rapid the rate of improvement, the steeper the curve; and, conversely, the more gradual the improvement, the flatter the curve. A 100% curve, indicating no improvement, would be horizontal. Portions of a curve which slope upward indicate a loss of production efficiency. It should be noted that the improvement curve is referred to by the complement of the rate of improvement and not by the rate itself. A 70% improvement curve, therefore, reflects a 30% rate of improvement.

d. An improvement curve can be expressed mathematically as well as graphically. The basic model is expressed by the equation:

$$y = ax^b$$

where y is the man-hours or cost to produce the xth unit. The parameter a represents the cost of the first unit and the parameter b indicates the rate of improvement. The relationship between b and the improvement curve percentage (P) is expressed by the following equations:

$$P = 100(2^b)$$

and

$$b = \frac{\log(P) - \log(100)}{\log(2)}$$

The curve shown in Figure F-1-1 was plotted from the equation

$$y = 100\ x\ -.321928$$

The value of -.321928 for b, was calculated as follows:

$$b = \frac{\log(80) - \log(100)}{\log(2)} = -.321928$$

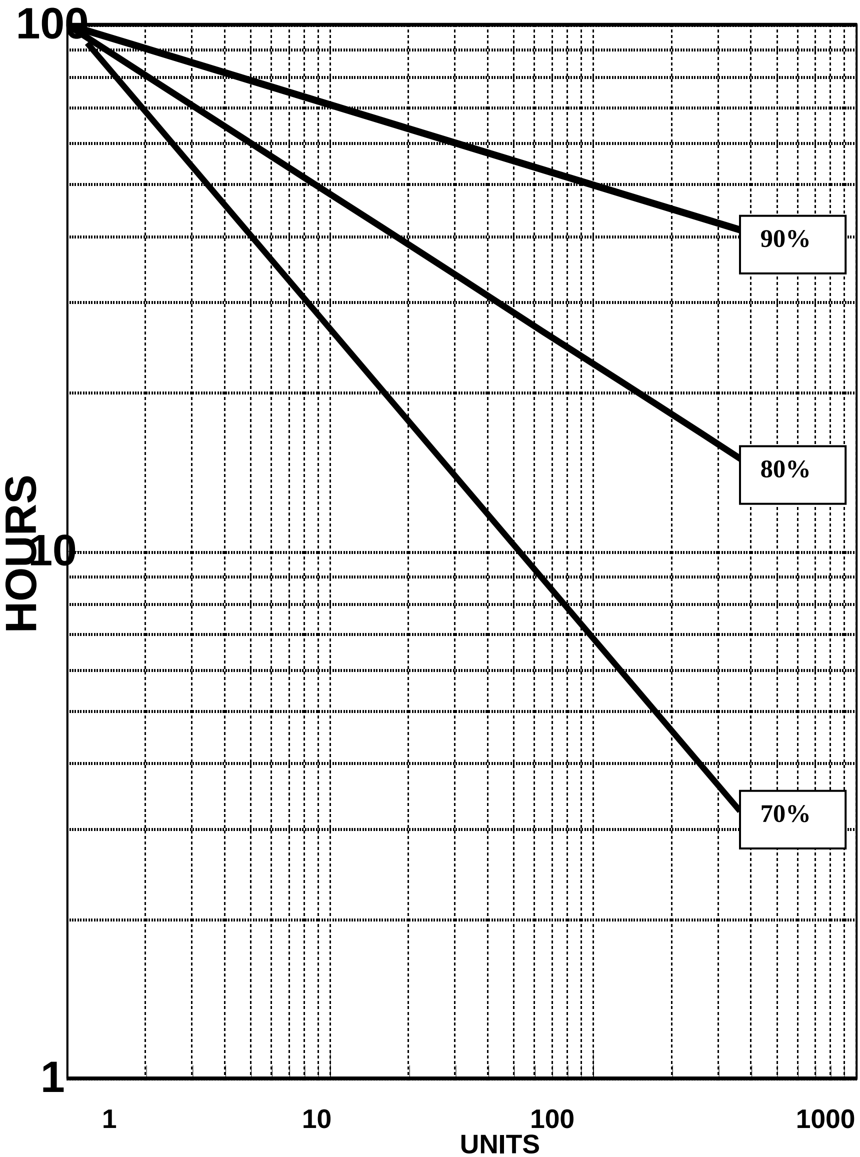
The basic improvement curve model can also be expressed by the following equation, which is obtained by taking the logarithms of both sides of the first equation,

$$y = ax^b\text{: } \log(y) = \log(a) + b\log(x)$$

It may be seen that this equation is in the same basic form as the simple linear regression equation which, as discussed in E-202.1, may be represented by a straight line on arithmetic graph paper. The only difference is that the logarithms of y, x, and a replace the values of y, x, and a in the simple linear model. The effect of using log-log paper is to convert the values of x and y to the logarithms of x and y, and it is for this reason that the improvement curve model becomes a straight line when plotted on log-log paper.

e. As illustrated in the above discussion and throughout this appendix, the improvement curve can be depicted both graphically and mathematically. Hence, projections of anticipated performance can be attained graphically by extending the line or by computation. While graphics facilitate analysis and presentation in audit reports, and are encouraged for these purposes, the mathematical approach provides more precise estimates and should be used to obtain estimates presented in audit opinions.

Figure F-1-3
Curves of Various Slopes on Logarithmic Paper



F-104 Fitting an Improvement Curve to Data

a. The data for the hypothetical improvement curve shown in Figure F-1-2 were selected to follow the improvement curve theory; that is, the values were calculated so that the cost-quantity relationship was perfectly linear for successive doubled quantities. As a result, the graph of that relationship on log-log paper is a straight line. In actual practice, a strictly linear relationship is seldom present; instead, the pattern is usually somewhat irregular. As a result, it is generally necessary to fit a line to the data as shown in Figure F-1-4.

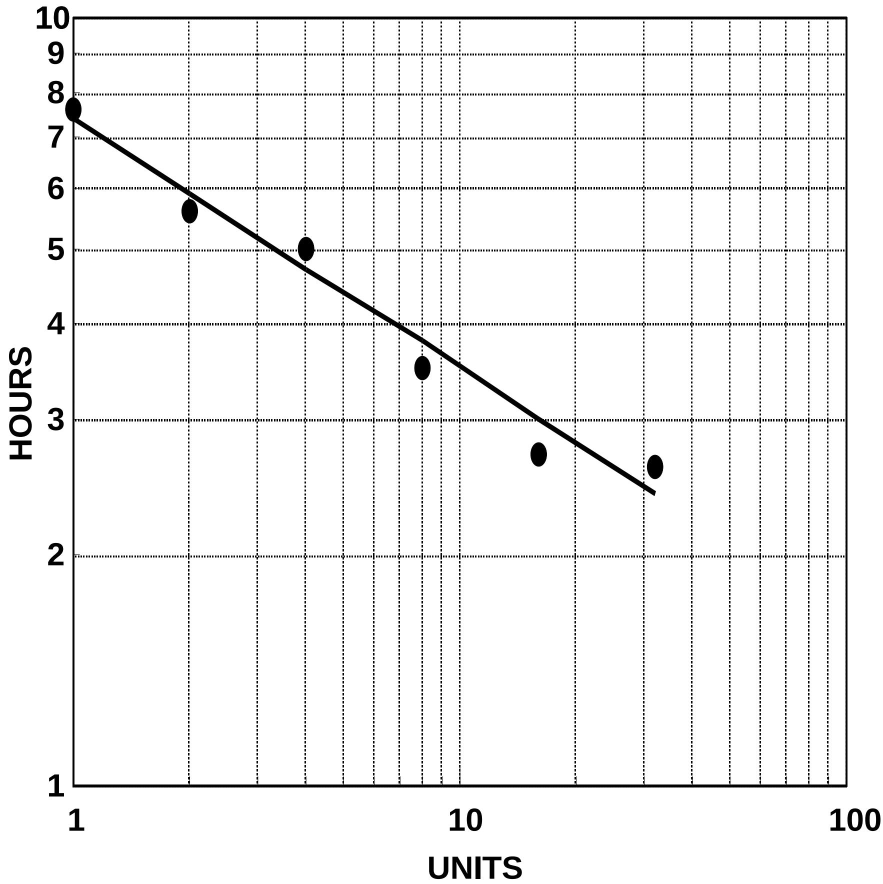
b. Before fitting a line to the data, the auditor must determine whether or not a clear trend exists. This can be determined by plotting the data on appropriate graph paper and reviewing the resultant diagrams. If the improvement curve theory is to be applied, the data pattern plotted on log-log paper should be approximately linear.

c. Frequently, the auditor may find that improvement curve assumptions are not valid in particular circumstances. For example the rate of cost reduction may not be constant, or it may be constant only for relatively short periods. In certain operations, unit production costs may reach a plateau where they may remain unchanged for a significant period of time or tend to vary in an erratic manner. Because the basic assumptions of the curve are not always valid, the auditor cannot assume their validity in any particular situation; to do so may lead to invalid conclusions.

d. When the preliminary study shows that the cost-quantity relationships are not sufficiently linear, no attempt should be made to apply the improvement curve techniques to the forecasting of costs. Instead, the auditor should use other analytical procedures such as those discussed in Appendix E. There are exceptions to this general prohibition. Data patterns that are otherwise approximately linear may contain significant variations. For example, engineering changes often result in such variations, or the data may be affected by breaks-in-production or the retention of learning from the manufacture of similar items. Methods of treating these types of cost fluctuations as part of the improvement curve theory are discussed in F-600.

e. When the cost-quantity relationships are sufficiently linear on a logarithmic graph to permit the application of the learning curve theory, an improvement curve can be fitted to the plotted data. The preferred and widely accepted method of fitting an improvement curve to data is the least-squares method discussed in Section 2, Appendix E. Computer software prepared for the purpose of providing least-squares fits of improvement curves to data are described in F-405.

Figure F-1-4
Typical Improvement Curve



F-105 Characteristics of the Improvement Curve

In comparing rates of improvement experienced by various contractors in the production of a common item, it may appear that a contractor operating on a 70% or 75% curve is doing a better job than one operating on a 90% or 95% curve. On the other hand, it may be contended that a high slope rate (a low improvement rate) is indicative of effective planning and efficient operation while a low percentage curve (a high rate of improvement) is indicative of poor planning and inefficient operation. The total contract cost, in any event, is the deciding factor in judging the economy of a particular operation. Effective planning and efficient operation from the start of a production cycle tends to keep costs at a relatively low level; but it does not follow, as will be shown later, that either low initial cost or operation on a low percentage curve will assure the lowest cost. From the standpoint of the improvement curve theory, three factors affect the total production run cost: (1) the slope of the improvement curve, (2) the level of costs at the start of operations, and (3) the length of the production run. These factors will be considered in the paragraphs which follow.

F-105.1 Slope

a. The improvement curve slope, expressed in percentage terms, is the primary characteristic by which a curve is identified. As previously discussed, it is actually the complement of the rate of cost reduction that occurs as production progresses. It is also the ratio (in percentage terms) of the unit cost of any given quantity to the unit cost of half of that same quantity. For example, the slope of the curve shown in Figure F-1-2 can be computed from the curve data given in F-103a. Using the hours for units 32 and 16 (32.77 and 40.97), the slope would be 32.77/40.97 or 80%.

b. The slope of an improvement curve also can be determined by graphical methods. The simplest of these methods involves the use of a protractor. The protractor is placed with its straight edge on a vertical line and the center of the straight edge on the intersection of the vertical line with the trend line. The angle between the two lines is measured in a counterclockwise direction from the vertical line to the trend line. The reading in degrees is divided by .9 to obtain the slope percentage. If a protractor is not available, another graphic method can be used. A line is first drawn parallel to the trend line and in such a position, either above or below the trend line, that it intersects the left-hand vertical (y) axis (the unit number one line) at the start of a vertical cycle (point A in Figure F-1-5). The units of the vertical scale may be read as percentages with 100% at the end of the cycle where the parallel line intersects the scale (point A). The percentage value of the point at which the parallel line intersects the vertical axis for unit two (point B) will be the slope of the curve (80% in the example). There are two methods by which the parallel line may be constructed: (1) by measurement, or (2) by use of a triangle and straight edge.

(1) To construct the parallel line by measurement, the trend line first should be extended, if necessary, to intersect the left-hand axis (point B in Figure F-1-6). The distance from this point to the start of the next vertical cycle (point A) is measured, and this distance (1 inch in the illustration) is laid out vertically from any second point on the trend line; in Figure F-1-6 this was arbitrarily done at unit 10 (point C). A straight line is then drawn from the point thus determined (point D) to the start of the vertical cycle (point A); this line is parallel to the improvement curve trend line, and the percentage value of the point where it crosses the unit two line indicates the slope of the curve.

(2) To draw the parallel line using a triangle and a straight edge, the triangle is placed on the graph with one edge (called the leading edge) lying along the trend line (position 1, Figure F-1-7). The straight edge is placed against the left-hand edge of the triangle; and the triangle is then moved to the left along the straight edge until its leading edge intersects the vertical axis at the start of a cycle (position 2). The parallel line is then drawn through this point along the leading edge of the triangle

(points x and y). The curve is thus determined to be a 76% curve (point y).

c. The degree of precision attainable by these methods is directly dependent on the skill and care exercised by the estimator, especially in constructing the curves and in reading values from the curves. For example, a small variation in placing a point or a line, or in reading a value from a trend line may have little significance, numerically, at the lower end of the logarithmic scale. At the upper end of the scale, however, the same physical amount of variation can have a significant effect even though the relative degree of error would be the same. This is evident from the fact that the distances between the values 2 and 4, 200 and 400, and 2,000 and 4,000 are equal on a logarithmic scale. Thus, a deviation that may appear small may represent a sizable variation in the upper end of the scale; even the width of a pencil line may make a material difference.

d. By using the above techniques in a reverse manner, the estimated cost of any subsequent unit or group of units may be computed (1) from the slope of the improvement curve and the known or estimated cost of any unit or lot of production, or (2) from the costs of any two units from the same production run.

(1) The validity of this procedure will be directly dependent on the validity of the underlying assumptions and the skill of the estimator. The cost-quantity relationships, both and in the future, must warrant the use of the improvement curve technique; and the assumed improvement curve rate must be valid for both the known cost and for the production throughout the forecast period. The auditor should assess the validity of these assumptions before applying this technique or before accepting any estimates based on its use.

(2) The technique of using the cost of a single unit or lot and the slope of the improvement curve as a base for estimating future costs is frequently used in pricing new or modified products. In these cases, the cost used may be the estimated cost of the first unit or lot and the improvement curve rate may be the average for the plant or for comparable previous production. In addition to computation by graphical means, the cost for doubled quantities could be computed by multiplication in the same manner that the table for Figure F-1-1 (F-103a.) was computed. The costs, however, usually will be computed by use of improvement curve tables or improvement curve software, as discussed in section 4.

F-105.2 The Vertical Position of the Improvement Curve and Length of the Production Run

For convenience, these two factors are discussed under one heading.

a. The commonly used index of the cost level or vertical position of the curve is the value assigned to the point where the improvement curve crosses the vertical (y) axis, the actual or calculated theoretical cost of the number one unit. In Figure F-1-8, the first or irregular portion of the solid line connects the first three plotted points of a typical set of improvement curve data. The straight portion of the line portrays the trend for units 4 through 80. The broken line extends this trend line backward to intersect the vertical axis at the 190 man-hour point. This number, the index of the vertical position of the curve, is referred to as the computed or theoretical value of the number one unit because it is the amount that the number one unit would have cost had the subsequent cost-quantity relationship existed from the start of production. In practice, the computed or theoretical cost of the number one unit often differs materially from the actual cost of the unit when a constant rate of improvement does not develop immediately. In the example in Figure F-1-8, this constant rate of improvement begins only with the fourth unit. Many factors can cause this difference and at times, as when the early production is performed in the pilot shop, two distinct curves may be apparent; for example, a steep one for the pilot shop production and a more shallow one for production in the regular shop.

b. The length of the production run (that is, the number of units or production lots of a particular product to be successively produced) frequently must be considered as a fixed factor in the evaluation of cost estimates for a particular contract, if production is to be confined to the requirements of that contract. However when there will be production for several contracts, Government or commercial, for the same or relatively the same product, evaluation must be made on the basis of the total production requirements.

c. As illustrated by curves A and B in Figure F-1-9, all curves which have the same slope are parallel. It follows, that for curves of the same slope, the one occupying the lowest position with reference to the vertical axis will yield the lowest total cost for any given quantity of production. The same general prediction may be made if the curves, though not parallel, are so positioned that they cannot cross (curves C and D) or will not cross within the production period for a particular quantity of product (curves E and F, projection period No. 1). However, if production were continued for curves E and F, they would eventually cross, and the cost of units produced thereafter and in time the total cost of all units produced would be lower for the curve with the higher starting cost and the lower slope rate (curve E, projection period No. 2). The three factors of slope, initial cost, and the number of units to be successively produced are interdependent and must be considered together in determining the overall economy of production operations. For any given number of units, however, the level of the curve with reference to the vertical axis and the slope of the curve determine the total cost.

Figure F-1-5
Determining Slope of a Trend Line for an
Improvement Curve on Logarithmic Paper

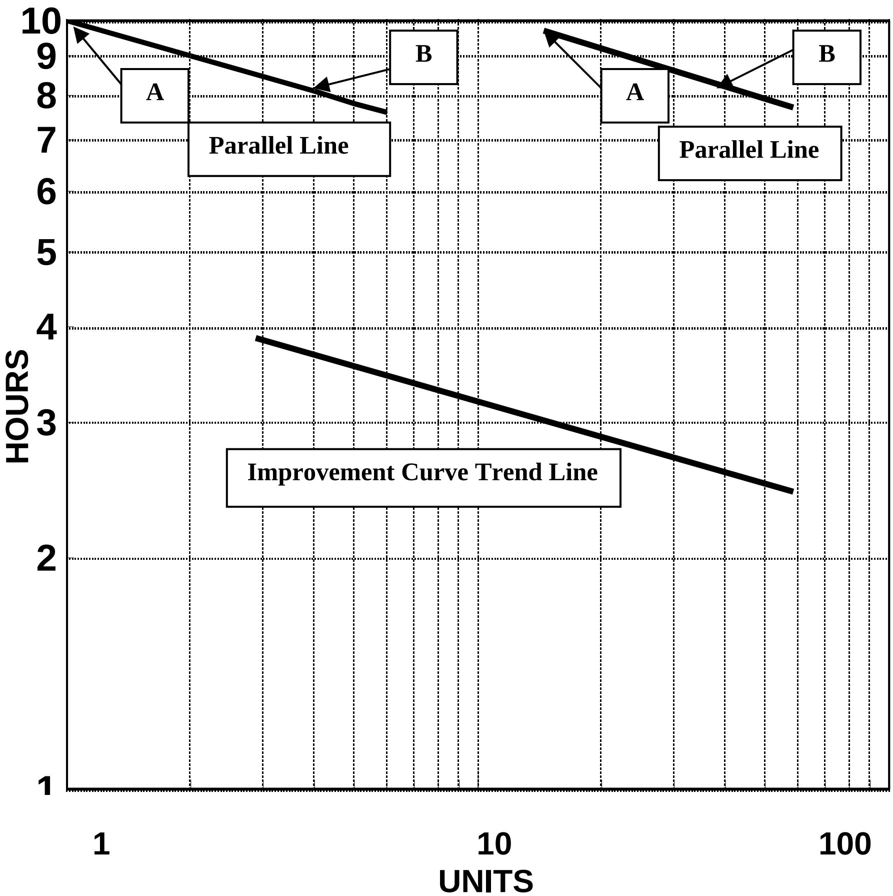


Figure F-1-6
Constructing a Parallel Line by Measurement

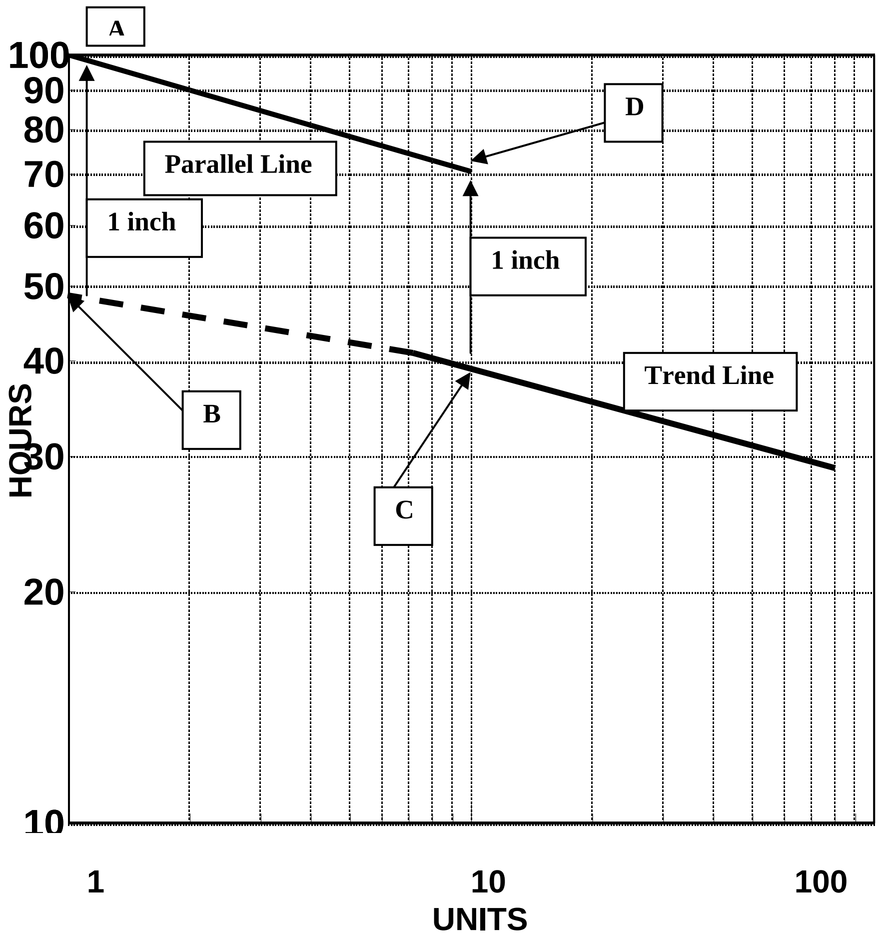


Figure F-1-7
Construction of Parallel Lines
Using Straight Edge and Triangle

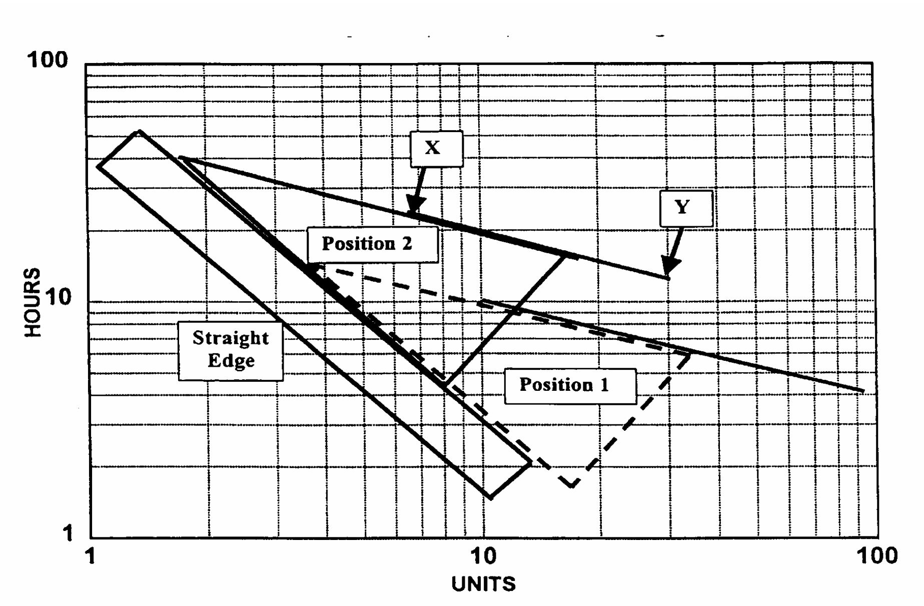


Figure F-1-8
Actual and Theoretical Values
of Number One Unit

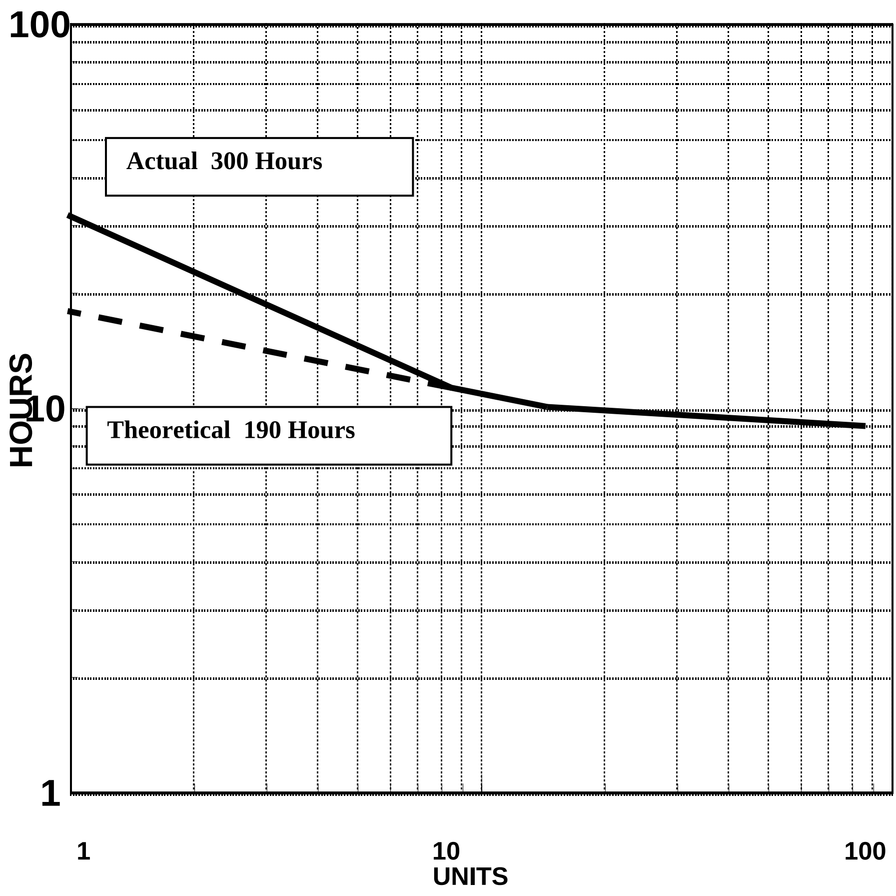
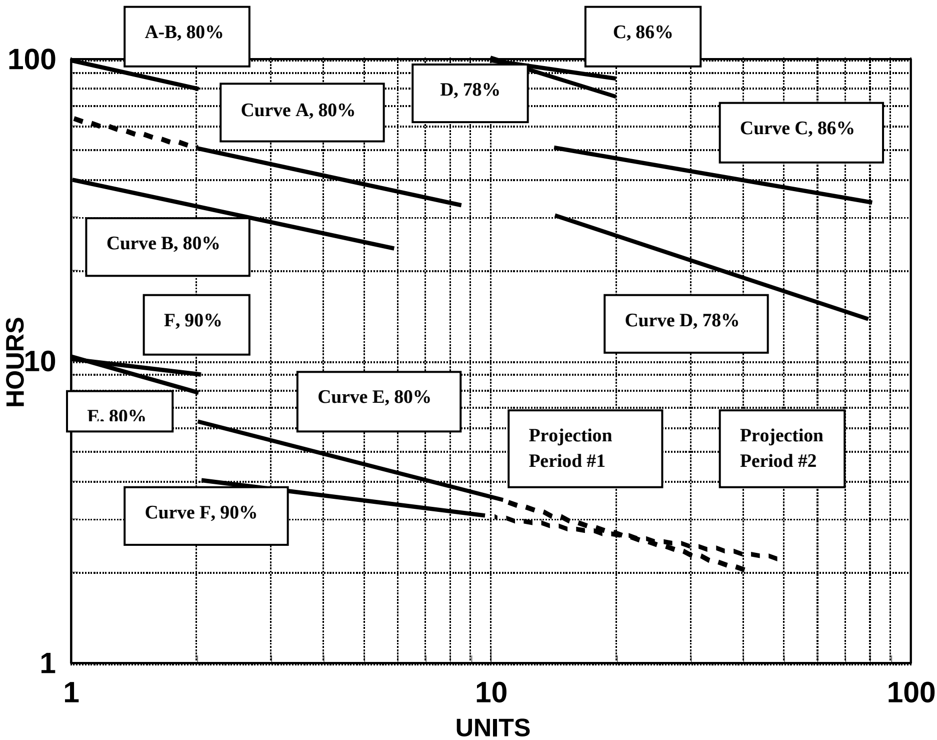


Figure F-1-9
Comparison of Curves -- Slope and Position



F-106 Significance of Vertical Position and Slope

a. A number of factors determine the overall level of costs and the rate of improvement and, thereby, the vertical position and slope of the curve. Whether a given change will affect the level or the slope, or both, is dependent not only on the type and extent of the change, but also and primarily on the timing and manner of its implementation. For example, improvements made in tooling, if introduced gradually during production, will tend to affect the slope of the curve, whereas a major change made at any one time could shift the subsequent cost level. A change in product design resulting in a major simplification of production can have a similar effect, reducing the subsequent cost level. Major changes are commonly made at the start of the production cycle, and their effect is reflected in the cost level of the number one unit. However, when major changes are introduced during production, the overall level of subsequent costs may change materially, as illustrated in Figure F-1-10. When this situation is encountered in the historical period, predictions of future costs should be based on the straight portion of the curve after the change. An attempt to fit an over-all trend line that would integrate the two sections of the curve would lead in most cases to incorrect cost estimates.

b. Factors which affect the slope of the curve were listed in paragraph F-102d. Some of these and others which also affect the vertical position of the curve are discussed below. Neither the listing of items nor the discussion of those listed is exhaustive, rather, the purpose of the discussion is to suggest avenues of investigation for determining the importance of any abnormalities in the cost trends. Possibly because the curve integrates the effects of so many operational factors, it frequently exhibits a considerable degree of stability.

(1) The relative amount of manual labor to machine time. In general, the higher the ratio of manual labor to machine time, the greater may be the rate of improvement and the smaller the slope value of the improvement curve, conversely, the higher the degree of automation, the less the opportunity for improvement by the individual operator. With the increased complexity and cost of automation, changes may tend to become less frequent and of relatively major significance. Thus, highly automated processes, after the shakedown period when improvement is rapid, may exhibit relatively little improvement for an extended period.

(2) The complexity of the product. Generally, as complexity increases, more man-hours are required and the vertical position of the curve becomes higher. Complexity also affords more opportunities for improvement and steeper slopes are typically encountered.

(3) The experience and skill of management and the work force. Inexperience usually results in higher first unit costs than would be expected with more experienced personnel, but it can also result in higher observed rates of improvement since the opportunities for improvement are greater. The fact that more experienced personnel might start at a lower first unit cost and proceed at a lower observed rate of improvement can be attributed to prior applicable experience gained elsewhere.

(4) Number of shifts and amount of overtime. Multi-shift operations and excessive overtime tend to reduce efficiency, with a resulting adverse effect on the vertical position of the curve.

(5) Plant capacity. Operation at other than optimum plant capacity may adversely affect the vertical position of the curve and, frequently, the slope.

(6) The costs of experimental models, prototypes, and pilot shop production. The cost of production in model and pilot shops tends to be higher than the cost of subsequent production in the regular shop. Whether or not the costs of these operations can be used, even with adjustment, as a basis for forecasting subsequent production costs in the regular production shop, can be determined only by careful analysis of the contractor's experience. Further, when these costs are used as a basis for developing a forecast, the auditor's report should include comments on this fact and all pertinent findings as to the historical reliability of this procedure. These prob-

lems arise primarily from a contemplated transfer of production from the model or pilot shop to the regular production facilities. They will not exist, as a rule, if production is to be completed in the model or pilot shop.

(7) The stability of the work force. When labor turnover is high, production efficiency and hence labor costs will be adversely affected. The vertical position of the curve usually will be higher than it otherwise would have been. The effect on the slope of the curve will depend on the rate of turnover and on the stability of that rate; for example, a consistent but excessive rate of turnover may substantially reduce overall improvement.

(8) Period between production units. When an extensive period elapses between the production of successive units, the rate of improvement will tend to be low because of the lapse of time between when a worker performs the identical operation on successive units. For example, the rates of improvement in the construction of large ships are generally less than in the manufacture of other end items which employ extensive manual labor but over much shorter periods of time.

(9) Documentation. Records should be maintained of problems encountered in past production and work methods, schedules, and layouts devised to alleviate these problems and improve future efficiency. In this manner, the lessons of the past will not be forgotten. If substantial periods elapse between the production of successive units, the extent of such documentation will affect the rate of improvement. When there is a break in production, the extent of such documentation will affect the amount of improvement retained when production is resumed.

(10) Engineering changes. Major engineering changes usually tend to disrupt normal improvement curve patterns. Because of the importance of these changes, they are separately discussed in F-502.

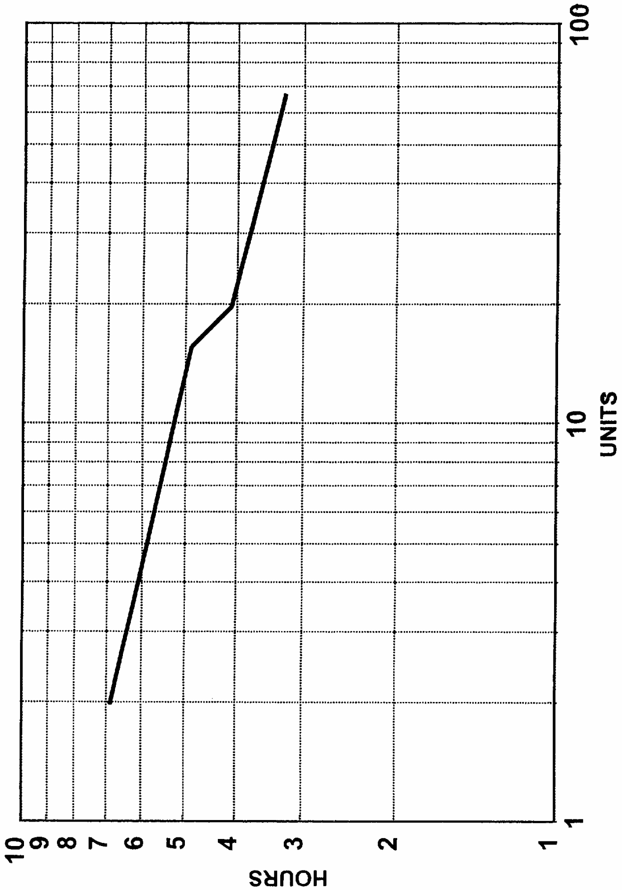
(11) Make-or-buy practices. Because a company's make-or-buy practices can have a significant effect on the work being performed, on the capacity level at which the company operates, and on unit labor requirements, they can also materially affect the vertical position and slope of the improvement curve. A major change during the production cycle can have an effect somewhat similar to an engineering change. A major change from make to buy may cause a rapid drop in cost, possibly followed by a change in slope. A major change from buy to make may cause a rapid increase in cost and can also be followed by a change in slope.

(12) Prior experience in producing similar items. If a contractor produces an item similar to items previously produced in the same facility with the same work force, it can be expected that some of the improvement gained on prior production will be carried over to the new item. The same effect is generally obtained when a contractor makes an item which has previously been produced by another contractor, and documentation on the lessons learned by the first contractor is made available to the second. In either case, it will appear that initial improvement is less than that experienced later in production, when the data are plotted normally. The appropriate adjustment is to reposition the data several units to the right in order to compensate for the number of units of retained improvement. This adjustment is discussed in F-503.

(13) Breaks-in-production. Resumption of production after an inordinate interval between units or lots will adversely affect the pattern of improvement and the vertical position of the curve can be expected to be higher than it was prior to the disruption. Breaks-in-production are discussed in some detail in F-504.

(14) Fluctuations in volume. When manloading requirements are not proportional to the number of units being produced, there will be instances when the volume of production impacts the production efficiency. A complete discussion of this factor is presented in F-505.

Figure F-1-10
Reduction in Cost Level
Resulting from Major Simplification



F-200 Section 2 --- Improvement Curve Types

F-201 Introduction

This section discusses and compares the two basic theories underlying improvement curves.

F-202 General

As has been previously stated, the basic concept of the improvement curve assumes that there will be a relatively constant rate of reduction in the unit cost for each successive doubling of the total production. This general concept has been expressed in two slightly different ways: (1) as the unit curve theory and (2) as the cumulative average curve theory. For each of these theories, two types of improvement curves may be constructed: a unit cost curve and a cumulative average cost curve. Thus, there are four improvement curves: a unit cost curve and a cumulative average cost curve for the unit curve theory, and a unit cost curve and a cumulative average cost curve for the cumulative average theory. The methods of constructing these four curves, their similarities and differences, and the difference between the two improvement curve theories, are discussed in the following paragraphs.

F-203 The Unit Curve Theory

The unit curve theory is based on the assumption that as production quantity is doubled from any level, the cost of the last unit of the doubled quantity is a constant percentage of the cost of the last unit before doubling. That is, the cost of the fourth unit is assumed to be the same percentage of cost of the second unit as is the cost of the eighth unit is to the cost of the fourth. This is the improvement curve theory discussed in section 1. It was illustrated with an 80% curve in Figure F-1-2. Based on the data given in the following table, an 80% unit cost curve and an 80% cumulative average cost curve computed under the unit curve theory are illustrated in Figure F-2-1.

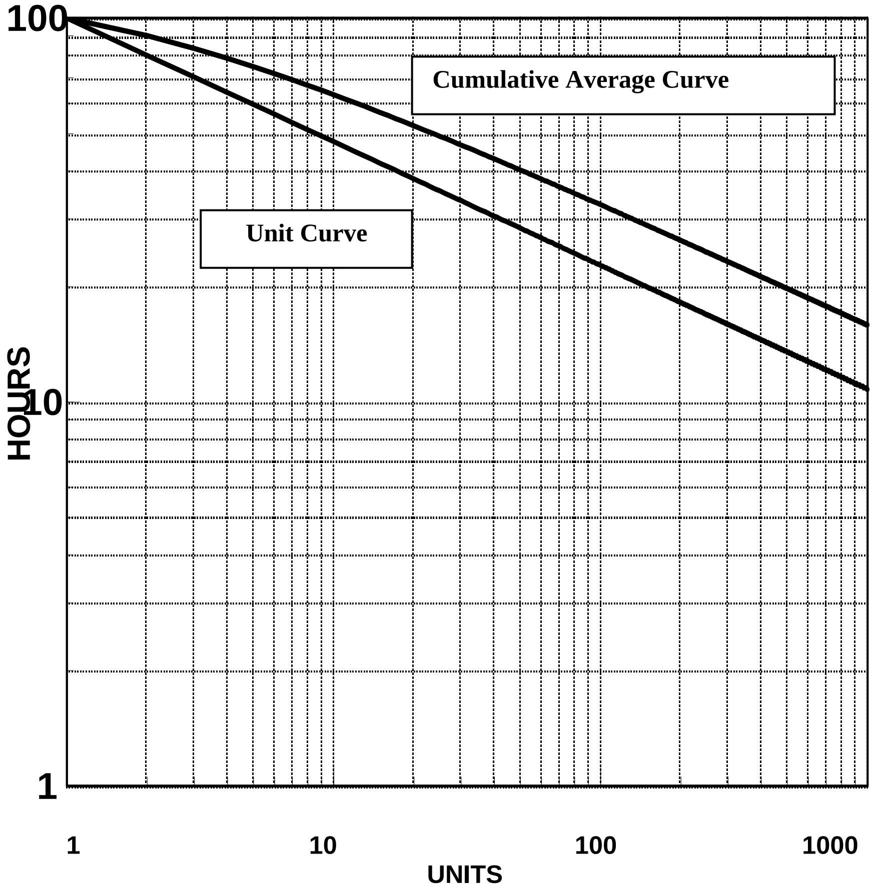
Table for Figure F-2-1
80% Unit Curve Theory

Unit No.	Unit Man-hours	Cumulative No. of Units	Cumulative Total Man-hours*	Cumulative Average Man-hours
1	100.00	1	100.00	100.00
2	80.00	2	180.00	90.00
4	64.00	4	314.21	78.55
8	51.20	8	534.59	66.82
16	40.96	16	892.01	55.75
32	32.77	32	1,467.86	45.87
64	26.21	64	2,392.45	37.38
100	22.71	100	3,265.08	32.65
1000	10.82	1000	15,867.09	15.87

* The totals include the values of omitted units

The data in columns 1 and 2 of this table are the same through unit 64 as the data given in the table for Figures F-1-1 and F-1-2 (F-103a.). As a result, the unit curve in Figure F-2-1 and the curve in Figure F-1-2 are identical. Because the assumed uniform rate of reduction applies to the cost of specific units, the unit cost line is linear. On the other hand, the cumulative average data, which are derived from the unit cost data, do not reflect a constant rate of improvement linear. However, the curvature in this line decreases rapidly, so that it becomes approximately linear and parallel to the unit cost line after the first 20 to 30 units.

Figure F-2-1
Unit Curve Theory (80% Curve)



F-204 The Cumulative Average Curve Theory

The hypothesis for the cumulative average curve theory assumes that as the total number of units successively produced is doubled, the cumulative average cost of each doubled quantity of production (that is, the average cost of units 1 and 2, of units 1 through 4, 1 through 8, 1 through 16, etc.) will decline by some constant percentage. The operation of this theory is illustrated in the following table and in Figure F-2-2.

Table for Figure F-2-2
80% (Cumulative Average Curve Theory)

Unit No.	Unit Man-hours	Cumulative No. of Units	Cumulative Total Man-hours*	Cumulative Average Man-hours
1	100.00	1	100.00	100.00
2	60.00	2	160.00	80.00
3	50.63	3	210.63	70.21
4	45.37	4	256.00	64.00
5	41.82	5	297.82	59.56
6	39.19	6	337.01	56.17
7	37.13	7	374.14	53.45
8	35.46	8	409.60	51.20
16	28.06	16	655.36	40.96
32	22.33	32	1,048.58	32.77
64	17.82	64	1,677.70	26.21
100	15.42	100	2,270.62	22.71
1000	7.34	1000	10,819.71	10.82

*Totals include values of omitted units.

The data in the last column of this table, the cumulative average man-hours, is identical with that in the unit man-hours columns in the table for Figure F-2-1 (F-203). Both tables reflect a 20% improvement for doubled quantities (80% curves). In Figure F-2-2 it is the cumulative average cost line which is linear, and the unit line which is curved. Again, the rate of curvature becomes negligible after the first few units and the unit cost line becomes approximately linear and parallel to the cumulative average cost line.

Figure F-2-2
Cumulative Average Curve Theory (80% Curve)

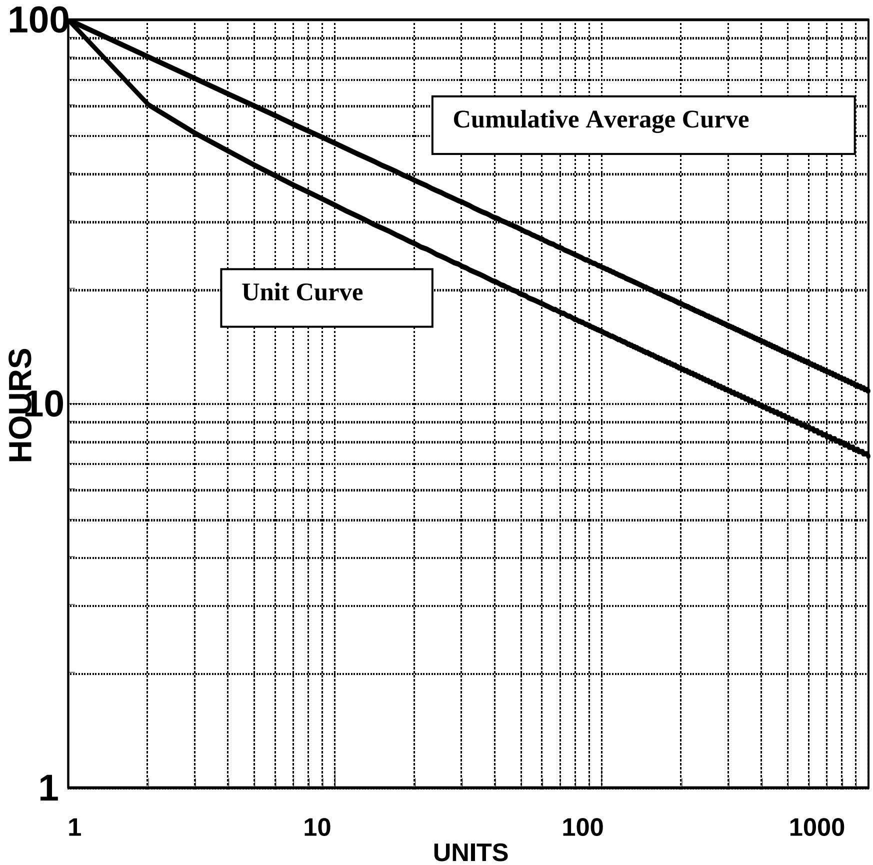
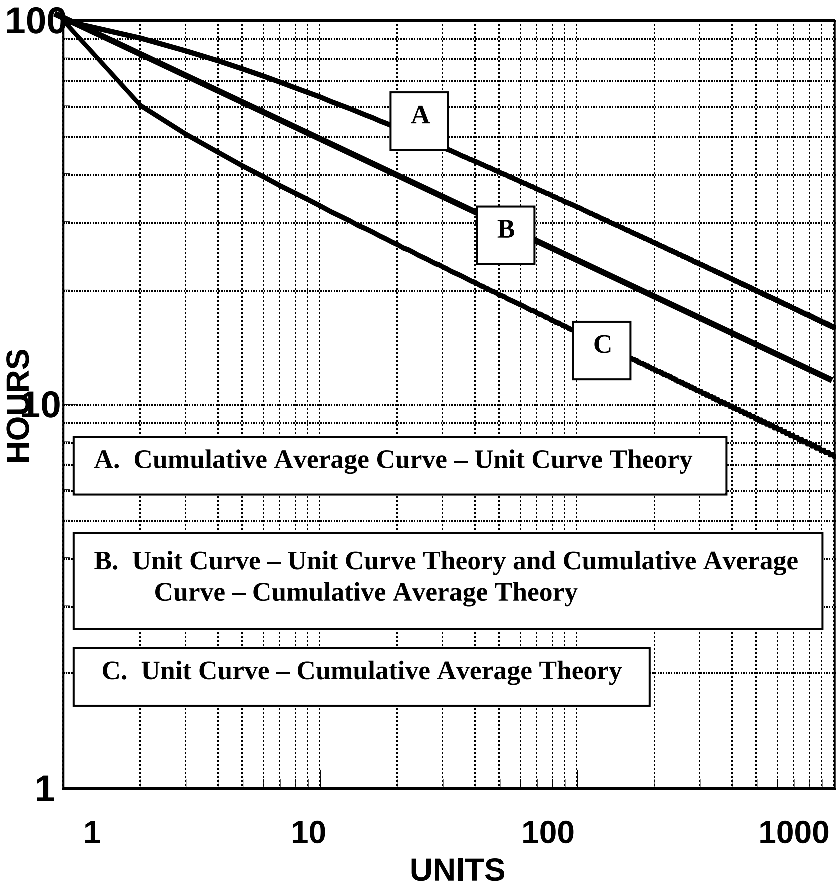


Figure F-2-3
Comparison of Unit and Cumulative Average Theories (80% Curve)



F-205 Comparison of Improvement Curve Theories

a. The auditor should understand from the discussion in the first part of this appendix that the improvement curve theory is not an expression of an exact or absolute principle. It is a generalization based on observed relationships between the production cost and the quantity produced which has been found to be sufficiently true to permit broad usage in the analysis and forecasting of product costs. The concept expresses an approximation, and many variations in the method of application have been developed to meet the needs and ideas of users. These differences represent varying methods of interpreting the general concept and the two theories of the improvement curve. The auditor must determine the appropriateness of the methods used by the contractor. He or she will not find it a difficult task if he or she understands the basic principles and interrelationships underlying and integrating the two slightly different expressions of the basic concept.

b. A comparison of the curves in Figure F-2-1 and F-2-2 will disclose a number of similarities and differences that are significant to their construction and interpretation. To facilitate this comparison the four curves have been combined in Figure F-2-3.

(1) It should be evident that the unit cost line for the unit curve theory and the cumulative average cost line for the cumulative average curve theory are identical for any given slope; an identity that is postulated in the two curve theories. As a result, these curves are represented by a single line in Figure F-2-3. Although the two lines are identical for any given slope, the basic data resulting in identical slopes is different. For example, if unit one requires 100 man-hours and unit two requires 80 manhours, under the unit curve theory this would be represented by an 80 percent unit curve, but under the cumulative average curve theory this would be represented by a 90 percent cumulative average curve, i.e., in order for the cumulative average curve under the cumulative average curve theory to be 80 percent, unit number two must require 60 man-hours.

(2) Except for the curvature in the first portion of the cumulative average line for the unit curve theory and in the first portion of the unit line for the cumulative average curve theory, which result primarily from the construction methods used, the only difference between curves of the same slope lies in the level or relative vertical positions of the unit and cumulative average lines. Regardless of the curve theory being followed, the cumulative average cost curve lies above the unit cost curve (occupies a higher position in relation to the vertical axis). This difference in position arises from the fact that each cumulative average cost is an average of all costs from the first unit through a given point of production and thus includes a portion of the high cost of the early, less efficient production; while the unit cost curve presents only the costs of individual units. However, because of the lack of linearity in the first part of the curves, the use of the cumulative average curve for the unit curve theory and of the unit curve for the cumulative average curve theory is not practical for forecasting the early cost of production. Beyond the first few units, however, the relationship between the unit and the cumulative average curves for either theory becomes relatively constant and either curve could be used.

(3) In practice, it is customary to plot the unit curve of the unit curve theory and the cumulative average curve of the cumulative average curve theory. This representation of the cumulative average curve can be misleading since the averaging process tends to smooth out the data pattern, concealing and suppressing significant deviations in unit costs. For this reason, the auditor should plot a unit curve as part of the analysis, regardless of whether the unit curve theory or the cumulative average curve theory is used.

c. Although the cumulative average theory was developed first, the unit curve theory is most commonly used. Furthermore, studies of Defense production data have generally provided more support for the unit curve theory. Accordingly, auditors should use the unit curve theory unless there is evidence that the contractor's experience has consistently followed the pattern predicated by the cumulative average curve theory.

F-300 Section 3 --- Fitting Improvement Curves to Lot Data

F-301 Introduction

This section discusses the application of the improvement curve theory to data accumulated in lots rather than by individual units.

F-302 General

a. The accounting systems maintained by the majority of manufacturers accumulate costs by production lots rather than by production units. Both the unit curve and the cumulative average curve theories can be applied to lot data.

b. Plotting a cumulative average curve presents no problem because the cumulative averages are simply plotted at the last unit number for each lot. However, in plotting a unit curve under either theory, it will be necessary to locate the unit number at which the average unit cost of each lot is plotted. This lot midpoint represents the unit number to which the average lot cost could be expected to apply if cost data were available on each unit in the lot. In consonance with the improvement curve theory, it may be assumed that the first unit a production lot would normally cost more to produce than the average for the lot and the last unit less. Some unit within the lot, however, would cost approximately the same as the lot average. The true lot midpoint will not usually be a whole number, although rule-of-thumb methods may result in whole numbers being used. For example, if a lot consists of units 1 and 2 on an 80% unit theory curve, the midpoint will obviously be between 1 and 2. Even if the lot consists of units 1 and 3, the midpoint will not be exactly 2 because the average unit cost of the first three units under an 80% unit theory curve is about 83.4% of the first unit cost, whereas the cost of the second unit is 80% (Table F-4-1). In general, a lot midpoint will be less than the mean of the first and last unit numbers in the lot. However, it will be quite close to the mean if (1) many units were produced in prior lots and/or (2) the slope of the curve is shallow.

c. Another consideration when an improvement curve is fitted to lot data is the weight to be given to each lot. Both theory and logic dictate that more weight be given to larger lots. Computer programs are available which provide the correct weighting of lots, as well as precise calculations of lot midpoints.

F-303 Locating Lot Midpoint by Rule-of-Thumb

Many short-cut methods of obtaining lot midpoints have been developed. The following rule-of-thumb is simple and fairly accurate in most applications: (1) for the first lot multiply the number of units by .3 and add one, (2) for all subsequent lots, divide the lot size by two and add the number of units in the preceding lots. An example of a unit curve theory graph constructed by this rule-of-thumb is presented in Figure F-3-1.

F-304 Computation of Lot Midpoints from Tables

Improvement curve tables (excerpts are shown in Tables F-4-1 and F-4-2) may be used to compute lot midpoints. This method is more precise than rule-of-thumb methods (F-303). The lot midpoint is determined in this method by finding the number of that unit within the lot whose factor is equal to the cumulative average factor for the lot. The method is illustrated below.

a. In this illustration it is assumed that the tables for an 80% curve under the unit curve theory (F-203) will be used and that the midpoint of a first lot of 10 units will be computed. From the cumulative total column of Table F-4-1 the cumulative average factor is found to be .631537 (6.315373 divided by 10). In the unit factor column, this falls between units 4 and 5. By interpolation, it is found that a unit factor of .631537 corresponds to unit no. 4.19. Therefore, 4.19 is considered the lot midpoint for a first lot of 10 units.

b. Table F-3-1 shows exact first lot midpoints for selected lot sizes and slopes. The table should not be used for second and subsequent lots.

Table for Figure F-3-1
Lot Average Cost Curve (Unit Curve Theory)

Lot No.	No. of Units in Lot	Cumulative No. of Units	Plotting Point	Lot Hours	Average Hours Per Unit
1	6	6	2.8	480	80
2	16	22	14	1,024	64
3	6	28	25	354	59
4	14	42	35	784	56
5	4	46	44	216	54

Figure F-3-1
Lot Average Cost Curve
(Unit Curve Theory)

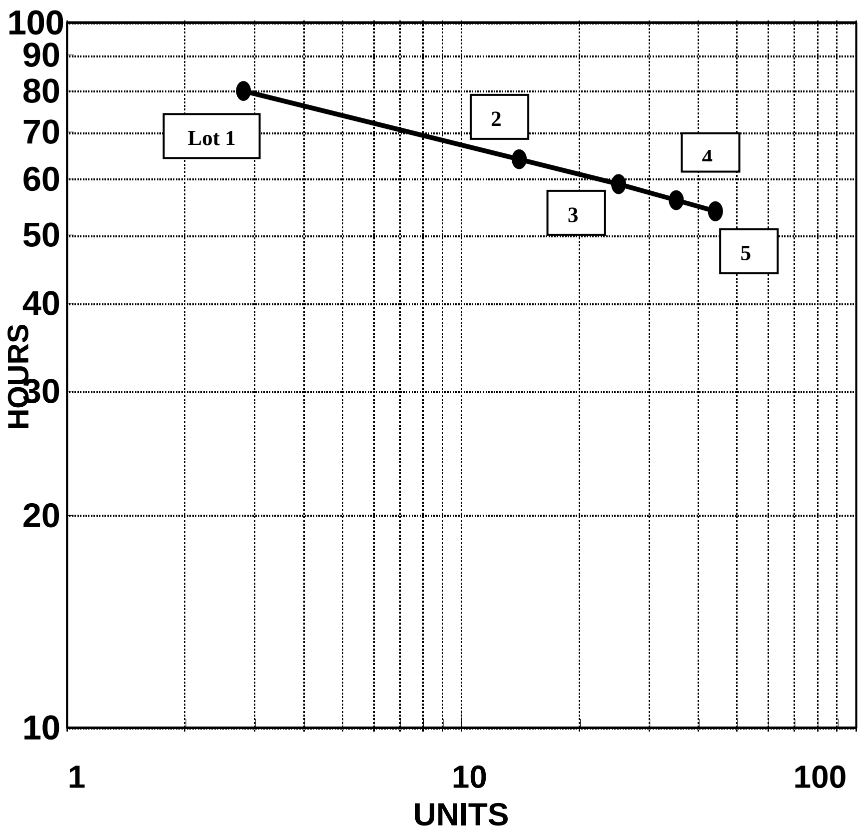


Table F-3-1
First Lot Midpoints -- Selected Quantities and Curves (Unit Curve Theory)

Units in 1st Lot	70% Curve	80% Curve	90% Curve
2	1.37	1.39	1.40
3	1.72	1.75	1.79
4	2.06	2.12	2.17
5	2.39	2.47	2.54
6	2.71	2.82	2.91
7	3.03	3.16	3.28
8	3.34	3.50	3.64
9	3.65	3.83	4.00
10	3.95	4.17	4.36
15	5.44	5.82	6.14
25	8.32	9.03	9.65
50	15.25	16.90	18.30
100	28.65	32.36	35.43
500	131.71	153.76	171.31
1000	258.15	304.43	340.67
10000	2495.48	3002.85	3384.18

F-305 Computer Programs and Lot Data

The preceding discussion is presented to provide a basic understanding of lot midpoints and to aid the auditor when computer facilities are not readily available. However, the preferred method of fitting improvement curves and projecting future costs is with the Agency's computer software designed specifically for this purpose. Unit and cumulative average improvement curve options are available on EZ-Quant software for curve estimation and cost projection.

F-306 Use of Equivalent Units

a. In some situations contractors may not segregate costs by units or lots. If, however, data are available on the actual labor hours charged to the production of an item during each month (or other period) and labor standards are used, it is often possible to determine the equivalent number of units produced during the period. The following tabulation illustrates computations for an item for which the standard hours per unit is 150.

Computation of Equivalent Units and Related Hours

Month Col. 1	Standard Hours Col. 2	Equivalent Units* Col. 3	Actual Hours Col. 4	Actual Hrs. Per Unit** Col. 5
Jan	6,037	40.24	8,624	214.31
Feb	17,058	113.72	15,972	140.45
Mar	36,307	242.05	26,728	110.42
Apr	48,973	326.49	27,851	85.30
May	51,207	341.38	26,528	77.71
Jun	51,853	345.69	25,630	74.14

*Col. 2 ÷ 150
**Col. 4 ÷ Col. 3

b. Even if labor standards are not used, this same general procedure can be followed if (1) records are maintained of the number of units in process at various stages of production at the end of each payroll period, and (2) there are estimates of the number of labor hours required for the work performed between stages. Such estimates could, for example, be obtained from the bill of labor submitted by the contractor in support of the bid proposal. To illustrate the procedure, suppose the contractor's records show the following status of production at the end of a payroll period:

	Number	Cumulative Total
Completed	100	100
In process through:		
Operation 3	10	110
Operation 2	30	140
Operation 1	20	160

The equivalent number of units produced through the end of the period could be calculated as follows:

Operation	Total Units Processed	Estimated Unit Labor Hours	Ex-tended Amount
1	160	10	1,600
2	140	30	4,200
3	110	40	4,400
4	100	20	2,000
Total		100	12,200

This would mean the equivalent of 122 units (12,200 – 100) had been produced through the end of the period. By applying this analysis to the status of operations at the end of each period, the equivalent number of units produced in each period can be obtained.

F-400 Section 4 --- Improvement Curve Techniques

F-401 Introduction

This section sets forth the methods of applying improvement curve theories.

F-402 General

As mentioned earlier, the methods or techniques of applying improvement curves are as follows:

a. The graphical method, in which the forecast values are derived from a graph upon which historical data have been plotted or one point is plotted and an improvement curve slope is drawn through the plot point. This method is satisfactory for exploratory purposes or where a high degree of accuracy is not required. Although this method is not desirable for expressing an audit opinion, inclusion of a graph in an audit report to depict the visual representation of the audit recommendation is desirable, and graphic analysis should always be utilized in conjunction with mathematical analysis.

b. The computational method, in which the forecast values are computed directly from the curve derived from the data. To eliminate the cumbersome procedure of manually computing projected costs, two methods of streamlined calculation are available: (1) tables of improvement curve factors and (2) the specially developed EZ-Quant computer software. The second option is the best method for both improvement curve estimation and cost projection. In addition to the significant savings in time and the superior accuracy of computer-based analysis, the computerized approach permits more complete and in-depth analysis than is possible by any other means.

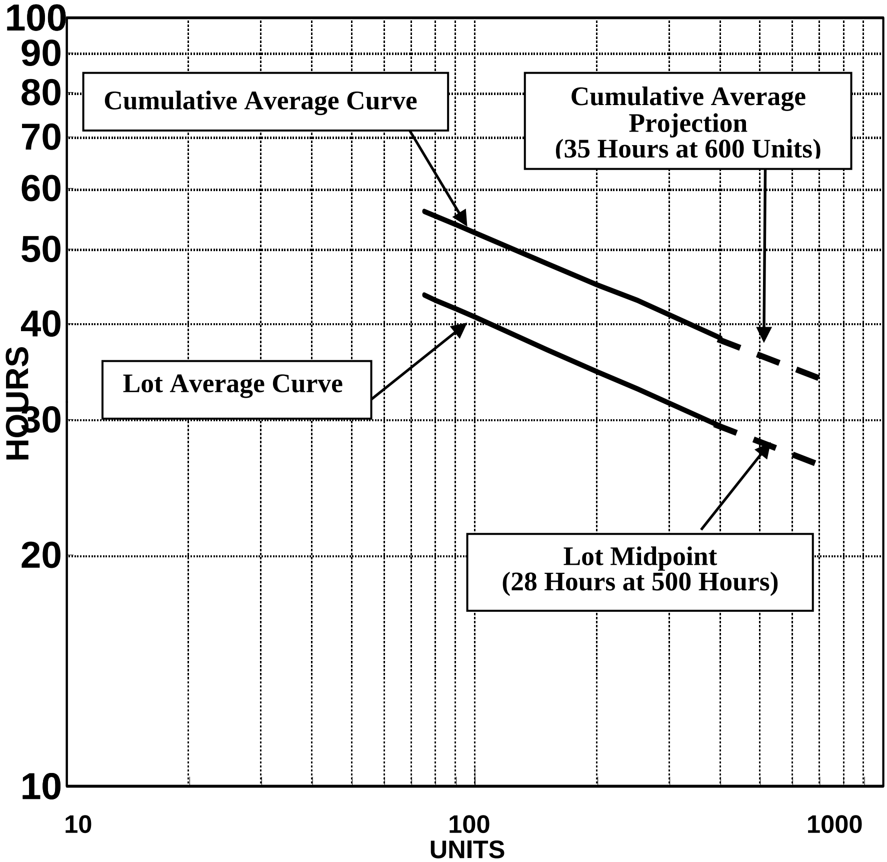
F-403 Graphical Projection

a. Figure F-4-1 illustrates the graphical method of estimating the hours required to produce units 401 through 600 of a given product. The production data upon which the projection is to be based is given in the table which follows:

Table for Figure F-4-1

Unit No.	Lot Average Unit Hours	Cumulative Total Hours From Unit 1	Cumulative Average Hours from Unit 1
75	43.6	4,199	56.0
80	42.9	4,415	55.2
90	41.8	4,838	53.8
100	40.8	5,250	52.5
150	37.0	7,187	47.9
200	34.6	8,975	44.9
250	32.9	10,660	42.9
400	29.4	15,308	38.3

Figure F-4-1
Cost Reduction



b. After plotting the lot average unit hour and cumulative average hour data, two trend lines were fitted to the plotted points. These lines were then extended to unit 600. The values necessary for estimating the labor-hour cost of units 401 through 600 may be read from either line. To use the lot average unit line, the 200 units (from 401 through 600) are treated as a single lot and the desired value is read from the midpoint of this lot. Using the rule-of-thumb, which assumes for second and subsequent lots that the midpoint is midway in a lot (F-303), the desired value is read at unit 500. This value, 28 hours, is the unit hour average for the lot; and the total hours required to produce the lot of 200 units is 200 x 28 or 5600 hours. To use the cumulative average curve, the auditor would proceed as follows:

(1) The cumulative average value of all units is read from the cumulative average curve at unit 600, the last unit in the lot; and this reading, 35 hours is multiplied by 600 to obtain the total cumulative hours, 21,000.

(2) The desired answer is the difference between the cumulative hours for the 600 and the 400 units, as shown in F-403a., (21,000 - 15,308) or 5,692 hours.

c. This graphical method has several disadvantages. Construction of the graphs with any degree of precision requires care and is time consuming, and the precision attained may be relatively low. More important, is the fact that the degree of precision is not known or predictable because it is largely dependent on the care and skill of the estimator.

F-404 Use of Improvement Curve Factors

a. Many contractors have developed tables of improvement curve factors which permit the computation of forecasted values to several more significant places than is possible by graphic means. Portions of two tables for both the unit curve theory and for the cumulative average curve theory are shown as Table F-4-1 and Table F-4-2.

b. Comparison of the tables developed by various companies will disclose a number of differences resulting, primarily, from the adaptation of the tables to the needs and specific methods of the developer. A principal difference is to be found in the unit number to which a value of unity is assigned. Several manufacturers, for example, assign the value of one to the number one unit. At least one assigned this value to unit 350 and another assigned it to unit 1,000. In each of these latter cases, the developers worked forward and backward from this point. Because of such differences, the values given for any particular unit in the different tables are not necessarily the same. Nevertheless, except for the very first units, the differences in the construction of the tables should not materially affect the answers to any given problem, provided that the instructions for using the tables are followed.

c. The tables are constructed on either the "unit curve theory" (F-203) or the "cumulative average theory" (F-204), depending on which concept the contractor believes will best meet his or her needs. The tables usually contain a series of factors for both the unit and cumulative average curves under the selected curve theory and for each percent of slope from 51% to 99%. The tables are bulky and to keep their size to a minimum, cumulative total factors, rather than cumulative average factors, are usually given. Cumulative average factors may be readily obtained by dividing the cumulative total factor by the corresponding cumulative number of units.

Table F-4-1
Improvement Curve Theory (Unit Curve Theory)

Unit Number	70% Curve		80% Curve		81% Curve	
	Unit	Cumulative Total	Unit	Cumulative Total	Unit	Cumulative Total
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	.700000	1.700000	.800000	1.800000	.810000	1.810000
3	.568180	2.268180	.702104	2.502104	.716065	2.526065
4	.490000	2.758180	.640000	3.142104	.656100	3.182165
5	.436846	3.195027	.595637	3.737741	.613068	3.795233
6	.397726	3.592753	.561683	4.299424	.580012	4.375245
7	.367397	3.960150	.534490	4.833914	.553458	4.928703
8	.343000	4.303150	.512000	5.345914	.531441	5.460144
9	.322829	4.625979	.492950	5.838863	.512748	5.972893
10	.305792	4.931771	.476510	6.315373	.496585	6.469478
15	.248208	6.273896	.418199	8.510537	.438996	8.767505
16	.240100	6.513996	.409600	8.920137	.430467	9.197970
20	.214055	7.406536	.381208	10.484043	.402234	10.846694
25	.190835	8.404015	.354784	12.308596	.375853	12.775653
32	.168070	9.643943	.327680	14.678620	.348678	15.292932
50	.133584	12.306881	.283827	20.121714	.304441	21.110529
64	.117649	14.050641	.262144	23.924477	.282430	25.199400
75	.108429	15.287445	.249095	26.727271	.269135	28.223858
100	.093509	17.790708	.227062	32.650811	.246597	34.641675
150	.075900	21.972246	.199276	43.233519	.217999	46.179489
200	.065456	25.482013	.181649	52.719963	.199743	56.585447
400	.045819	36.259632	.145319	84.848727	.161792	92.153287
800	.032074	51.355161	.116256	136.269264	.131052	149.788541

Unit Number	82% Curve		85% Curve		90% Curve	
	Unit	Cumulative Total	Unit	Cumulative Total	Unit	Cumulative Total
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	.820000	1.820000	.850000	1.850000	.900000	1.900000
3	.730127	2.550127	.772915	2.622915	.846206	2.746206
4	.672400	3.222527	.722500	3.345415	.810000	3.556206
5	.630786	3.853313	.685671	4.031086	.782987	4.339193
6	.598704	4.452016	.656978	4.688064	.761585	5.100778
7	.572855	5.024872	.633656	5.321720	.743948	5.844726
8	.551368	5.576240	.614125	5.935845	.729000	6.573726
9	.533085	6.109324	.597397	6.533242	.716065	7.289790
10	.517244	6.626569	.582820	7.116063	.704688	7.994479
15	.460554	9.030936	.529965	9.861056	.662568	11.383717
16	.452122	9.483058	.522006	10.383062	.656100	12.039187
20	.424140	11.219085	.495397	12.402277	.634219	14.607759
25	.397891	13.258028	.470145	14.800727	.613068	17.713230
32	.370740	15.929740	.443705	17.981384	.590490	21.910730
50	.326270	22.142563	.399623	25.513111	.551761	32.141956
64	.304007	26.535175	.377150	30.931342	.531441	39.707512
75	.290511	29.795770	.363382	34.994932	.518782	45.475307
100	.267542	36.742290	.339680	43.753867	.496585	58.141020
150	.238219	49.308133	.308875	59.888286	.466904	82.155823
200	.219384	60.709807	.288728	74.788513	.446927	104.964061
400	.179895	100.036233	.245419	127.569018	.402234	189.267825
800	.147514	164.547743	.208606	217.314261	.362011	341.034718

Table F-4-2
Improvement Curve Theory
(Cumulative Average Curve Theory)

Unit Number	70% Curve		80% Curve		81% Curve	
	Unit	Cumulative Total	Unit	Cumulative Total	Unit	Cumulative Total
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	.400000	1.400000	.600000	1.600000	.620000	1.620000
3	.304541	1.704541	.506311	2.106311	.528194	2.148194
4	.255459	1.960000	.453689	2.560000	.476206	2.624400
5	.224232	2.184232	.418187	2.978187	.440941	3.065341
6	.202125	2.386357	.391911	3.370098	.414733	3.480074
7	.185419	2.571777	.371329	3.741427	.394135	3.874209
8	.172223	2.744000	.354573	4.096000	.377319	4.251528
9	.161460	2.905460	.340546	4.436546	.363208	4.614736
10	.152465	3.057925	.328552	4.765099	.351116	4.965852
15	.122626	3.723113	.286705	6.272988	.308728	6.584946
16	.118487	3.841600	.280612	6.553600	.302529	6.887475
20	.105279	4.281095	.260614	7.624158	.282128	8.044681
25	.093609	4.770870	.242146	8.869596	.263210	9.396316
32	.082252	5.378240	.223324	10.485760	.243847	11.157710
50	.065183	6.679218	.193080	14.191354	.212539	15.222031
64	.057341	7.529536	.178203	16.777216	.197039	18.075490
75	.052816	8.132144	.169269	18.682129	.187698	20.185106
100	.045509	9.350905	.154213	22.706166	.171892	24.659691
150	.036907	11.385001	.135269	29.891406	.151880	32.699872
200	.031815	13.091267	.123271	36.329866	.139126	39.948699
400	.022256	18.327774	.098577	58.127785	.112649	64.716892
800	.015574	25.658884	.078846	93.004456	.091229	104.841366

Unit Number	82% Curve		85% Curve		90% Curve	
	Unit	Cumulative Total	Unit	Cumulative Total	Unit	Cumulative Total
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	.640000	1.640000	.700000	1.700000	.800000	1.800000
3	.550380	2.190380	.618745	2.318745	.738618	2.538618
4	.499220	2.689600	.571255	2.890000	.701382	3.240000
5	.464329	3.153929	.538355	3.428355	.674934	3.914934
6	.438294	3.592223	.513510	3.941866	.654579	4.569512
7	.417764	4.009987	.493730	4.435595	.638123	5.207635
8	.400957	4.410944	.477405	4.913000	.624365	5.832000
9	.386820	4.797764	.463576	5.376576	.612581	6.444581
10	.374680	5.172444	.451628	5.828204	.602299	7.046880
15	.331925	6.908304	.408968	7.949480	.564778	9.938521
16	.325645	7.233948	.402620	8.352100	.559079	10.497600
20	.304922	8.482809	.381514	9.907947	.539900	12.684385
25	.285628	9.947271	.361628	11.753620	.521485	15.326705
32	.265795	11.863675	.340933	14.198570	.501937	18.895680
50	.233530	16.313524	.306649	19.981154	.468609	27.588069
64	.217457	19.456427	.289253	24.137569	.451199	34.012224
75	.207734	21.788301	.278619	27.253642	.440374	38.908649
100	.191218	26.754179	.260343	33.967962	.421424	49.658525
150	.170178	35.732810	.236640	46.331192	.396134	70.035568
200	.156686	43.876854	.221161	57.745536	.379137	89.385344
400	.128436	71.958040	.187932	98.167411	.341158	160.893620
800	.105299	118.011186	.159718	166.884598	.307013	289.608516

F-405 EZ-Quant Computer Programs

Comprehensive guidance on the use of EZ-Quant is given in the software's "help" documentation. EZ-Quant includes the following improvement curve models:

a. Estimated least squares curve fits to data using the unit curve theory and the cumulative average theory models.

b. Models to project values on an improvement curve defined by a percentage slope and the cost of any unit or lot.

c. Special application improvement curve models which account for engineering design changes, production breaks, retained prior improvement, or variations in production rates. These additional procedures are discussed in section 5.

F-406 Coefficient of Determination

a. The least squares improvement curves fitted to the data by the EZ-Quant options generally satisfy the requirements of regression analysis. Accordingly, the coefficient of determination (r-squared) is included in the output. This statistic measures the extent to which variations in unit costs can be explained by difference in unit numbers. Paragraph E-205 discussed correlation analysis and provides guidance on interpreting the index of determination and determining the existence of correlation. In using Table E-2-1, it should be noted that mathematical models for the standard unit curve theory and cumulative average have two parameters. However, each of the more advanced models discussed in F-503, F-504, and F-505 has three.

b. Certain improvement curve options of EZ-Quant determine the comparison assurance (or confidence) that is associated with the coefficient of determination for the regression equation. This correlation analysis statistic is discussed in paragraph E-205.2.

F-407 Selection of a Curve

The best possible source of improvement curve data is the records of the contractor who is to produce an item. If the contractor has produced the same item in the past, its records can usually be used to obtain both the percentage slope and the theoretical first unit. Even if the contractor has not produced the item before, its experience in producing other items at the facilities planned for the new item will generally provide a more reliable percentage than the experience from another contractor. It should also be noted that while improvement curves can best be fitted to direct labor hours or costs which have been segregated by unit or lot, it is often possible to develop satisfactory improvement curves from monthly or weekly costs and equivalent units of production (F-306), or even from cost recorded against successive contracts.

F-500 Section 5 --- Other Factors Which Affect Improvement

F-501 Introduction

This section describes factors which may cause departures from normal improvement curve patterns. It also discusses methods of measuring and compensating for the effects of these factors.

F-502 Engineering and Other Major Changes

a. Changes in a product and in the method of its manufacture will affect the unit cost of the product and, therefore, the slope and vertical position of the improvement curve. Most of these changes are relatively minor; and, because they are constantly taking place, they form a continuing and repetitive pattern of change. Their combined and continuing impact on product unit cost is one of the principal improvement factors that the curve is designed to measure. The use of the improvement curve to measure a rate of change is a dynamic method of analyzing costs. Where other methods assume a constancy in composition of a product and in the technology of its production, the theory of the improvement curve assumes the constancy of change. It assumes that the rate of change is the factor that will be constant. To the extent that this assumption is true, the curve, appropriately plotted, will be linear; and the slope of the curve will be constant.

b. It must be recognized that these improvement curve assumptions encompass only those changes which compose the normal, continuing, repetitive pattern of change. There are, however, other changes, occasional in frequency, that have an abrupt and major impact on unit costs. These changes tend to produce a sharp and material deviation in the slope and vertical position of the improvement curve, as shown between lots 5 and 6 in Figure F-5-1. Major changes in the design of a product, commonly known as engineering changes, are one of the most common causes of these sharp deviations in the level and trend of the curve. There are, however, a number of other causes which can have the same or a similar effect, such as a major change in tooling and equipment, a major shift towards automation, or the production of a major component previously purchased.

c. The difficulty of forecasting from a curve that reflects an engineering or other major change may be seen from the example given in Figure F-5-1. In this example, a major change was made in a component in lot 6. As a result, a sharp rise in the vertical position of the curve occurred between lots 5 and 6. Though the curve slopes back sharply thereafter, it does not begin to reflect a stabilized trend until lot 10, when it becomes asymptotic to the trend before the change. Projection of the basic trend at lot 6 to forecast lots 7 to 10 would be meaningless, as would a projection of any segment of the line connecting any of these points.

d. The first step in appraising the full impact of engineering changes should be the segregation of costs between components affected by the change and those not affected. When unit cost or lot costs by components are available, there is usually no problem in securing this segregation. When appropriate data is not readily available, the auditor may segregate costs on some other basis provided it measures the relative effort in the changed portion to that in the whole product.

Asymptote: A curve always approaching but never meeting a straight line; tangent at infinity.

Table for Figure F-5-1						
Lot No.	Units Per Lot	Total Units	Lot Mid-Point	Lot Plotting Point	Total Hours Per Lot	Average Hours Per Lot
1	10	10	4.0	4.0	2000	200
2	10	20	5.0	15.0	1100	110
3	20	40	10.0	30.0	1600	80
4	20	60	10.0	50.0	1240	62
5	20	80	10.0	70.0	1080	54
6	8	88	4.0	84.0	720	90
7	12	100	6.0	94.0	840	70
8	30	130	15.0	115.0	1680	56
9	40	170	20.0	150.0	1840	46
10	40	210	20.0	190.0	1600	40
11	80	290	40.0	250.0	2800	35
12	80	370	40.0	330.0	2400	30
13	80	450	40.0	410.0	2160	27
14	120	570	60.0	510.0	2880	24
15	120	690	60.0	630.0	2640	22
16	120	810	60.0	750.0	2400	20

e. The data for a simplified example showing the plotting of separate curves for the changes and unchanged components are shown in the following table and graphically displayed in Figures F-5-2 and F-5-3. Curve A in Figure F-5-2 reflects the impact of an engineering change in Component A. Figure F-5-3 reflects an uninterrupted and uniform rate of improvement for all other components. Figure F-5-3 therefore presents no special evaluation problem, but the changed segment of Curve A should be replotted as though it were a new unit, as shown in Curve B of Figure F-5-2. It may be seen in Figure F-5-2 that Curve B curves downward slightly. This indicates there is some retention of learning in the production of Component A. Use of the retained prior improvement curve option of EZ-Quant as described in F-503, indicates that this retained improvement (or learning) is equivalent to the production of 5 units. Accordingly, the data should be replotted at unit nos. 9, 19, 40, etc. By computing and then combining separate evaluations for Component A, and for all other components, an evaluation for the entire product may be obtained.

f. When unit or lot costs for the changed work are not available, it may be possible to obtain a reliable engineering estimate of the proportion of direct labor for the lots prior to the change which is accounted for by the unchanged work. This proportion is applied to the hours for each lot prior to the change. The improvement curve is fitted to the resultant data to estimate the proportion of labor hours in subsequent lots which are related to the unchanged work. This portion is deducted from the total hours per unit for these subsequent lots to obtain estimates of the hours applicable to the changed work. If, in the foregoing example, an estimated 70% of the work prior to the change related to unchanged operations, the following analysis would be made:

Average Hours per Lot
Unchanged Operations

Lot No. Col. 1	Total Col. 2	Historic Col. 3	Projected Col. 4	Changed Col. 5
1	200	140		
2	110	77		
3	80	56		
4	62	43.4		
5	54	37.8		
6	90		34.49	55.51
7	70		32.76	37.24
8	56		29.91	26.09
9	46		26.47	19.53
10	40		23.73	16.27
11	35		20.95	14.05
12	30		18.41	11.59
13	27		16.65	10.35
14	24		15.07	8.93
15	22		13.66	8.34
16	20		12.60	7.40

Table for Figures F-5-2 and F-5-3
ENGINEERING CHANGES
Component A and All Other Components

				Component A Replotted As		Average Hours Per Lot		
Lot No.	Units Per Lot	Lot Mid-Point	Lot Plotting Point	Lot No.	At Unit No.	Component A	All Other Components	Total
1	10	4.0	4.0			70.0	130.0	200.0
2	10	5.0	15.0			44.0	66.0	110.0
3	20	10.0	30.0			32.0	48.0	80.0
4	20	10.0	50.0			24.0	38.0	62.0
5	20	10.0	70.0			20.0	34.0	54.0
6	8	4.0	84.0	1	3.4	59.0	31.0	90.0
7	12	6.0	94.0	2	14	40.0	30.0	70.0
8	30	15.0	115.0	3	35	29.0	27.0	56.0
9	40	10.0	150.0	4	70	22.0	24.0	46.0
10	40	20.0	190.0	5	110	18.0	22.0	40.0
11	80	40.0	250.0	6	170	15.4	19.6	35.0
12	80	40.0	330.0	7	250	12.8	17.2	30.0
13	80	40.0	410.0	8	330	11.2	15.8	27.0
14	120	60.0	510.0	9	430	10.0	14.0	24.0
15	120	60.0	630.0	10	550	9.0	13.0	22.0
16	120	60.0	750.0	11	670	8.0	12.0	20.0

Figure F-5-1
Engineering Changes

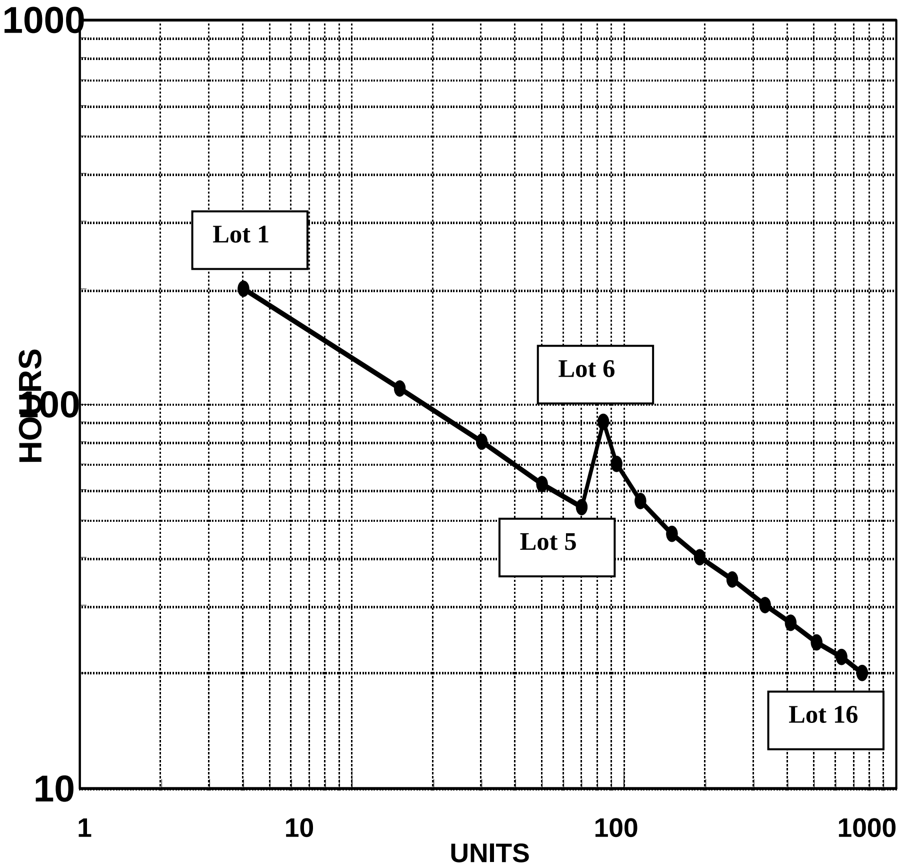


Figure F-5-2
Engineering Changes -- Component A

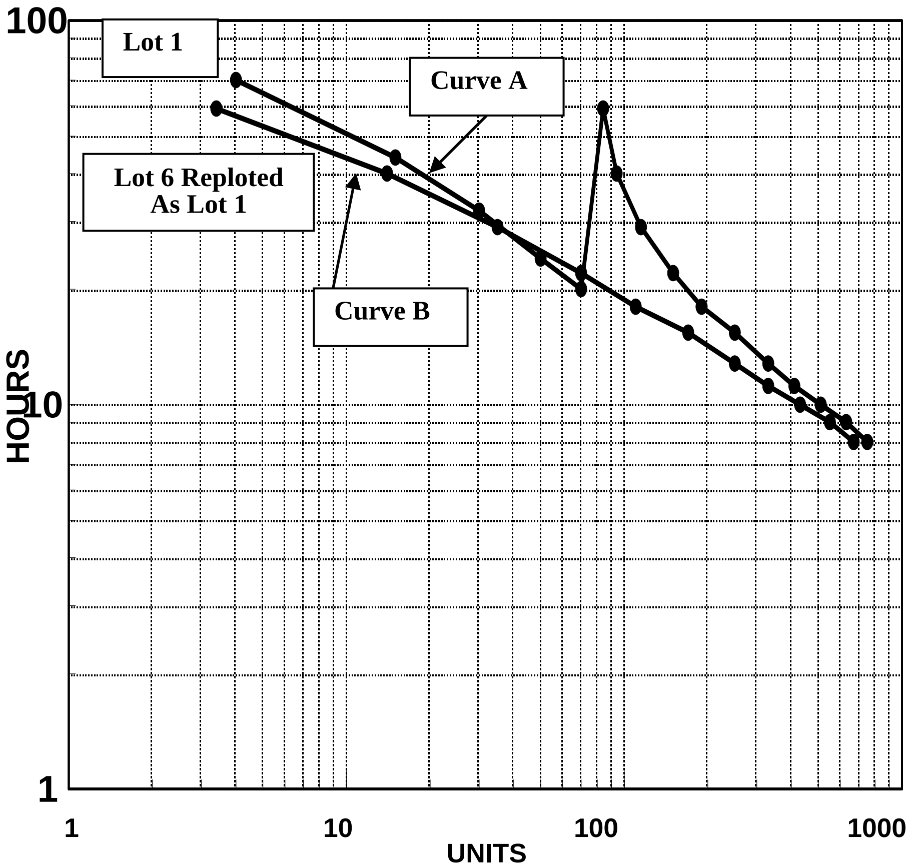
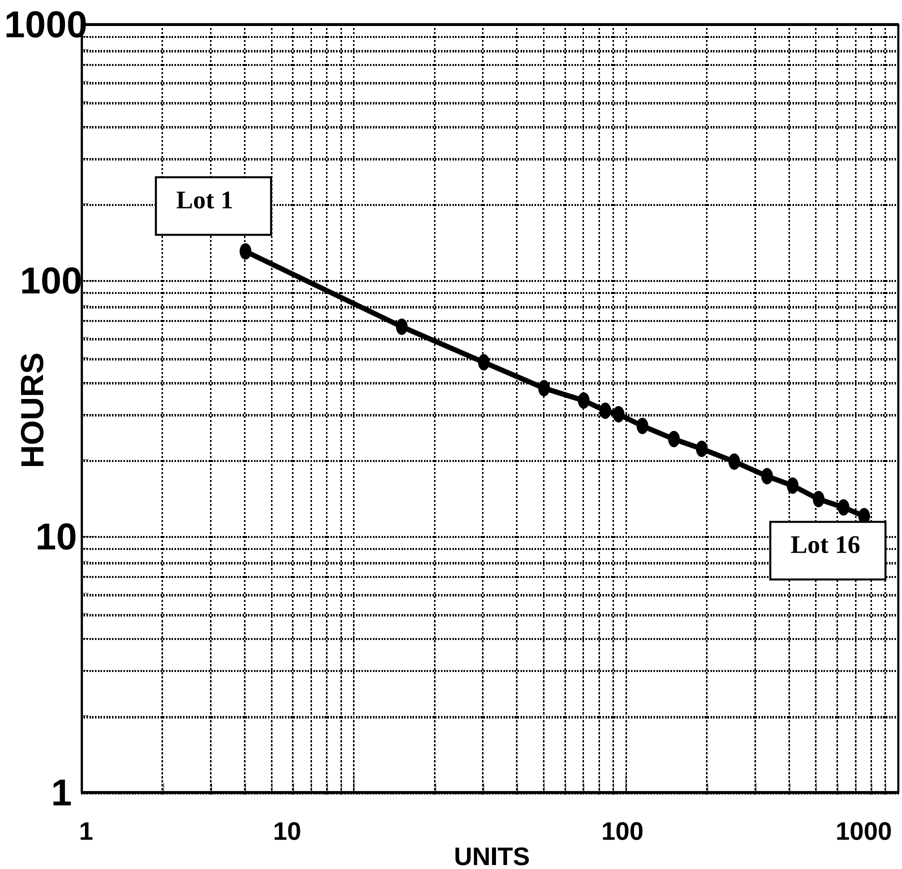


Figure F-5-3
Engineering Changes -- All Other Components



In this example, column 3 equals 70% of column 2. A weighted least squares fit to the data in column 3 resulted in an improvement curve with a slope of 72.644% and a theoretical first unit cost of 266.68. Projections along the curve produced the estimate of 34.49 for units 81 to 88, 32.76 for units 89 to 100, etc., shown in column 4. Column 5 is the difference between columns 2 and 4. It corresponds to the column headed "Component A" in the table referred to in the preceding subparagraph e and could be used in the same manner to project future costs of the changed work.

g. The design change improvement curve model of EZ-Quant estimates curve segments for:

(1) the pre-change production history,

(2) the unchanged portion (remaining original work), and

(3) the changed portion (or new work).

The procedure also computes projected cost or hours for user-specified lots.

F-503 Measuring Retained Prior Improvement

a. Where a contractor produces a new item that is similar to items produced in the past, it is likely that the first unit of the new item will require less cost than if the similar item had not been produced. In other words, prior improvement (or learning) achieved on the earlier item may be retained, thereby benefiting the new item. For example, if the retained prior improvement was equivalent to the production of 10 units, the labor-hour requirement for the first unit of the new model would correspond to unit 11 on a normal improvement curve, the hours required for the second unit would correspond to unit 12.

b. Typically, where retained prior improvement is a factor in determining costs required to produce an item, the trend line of actual costs will be "humped" as illustrated by the points connected by the solid line in Figure F-5-4. Moving the points five units to the right produces the approximately linear pattern shown by the x's. This pattern is readily discernible through an evaluation of the "percent difference" statistics. If the first two or three data points are below the computed improvement curve, this may indicate retained prior improvement.

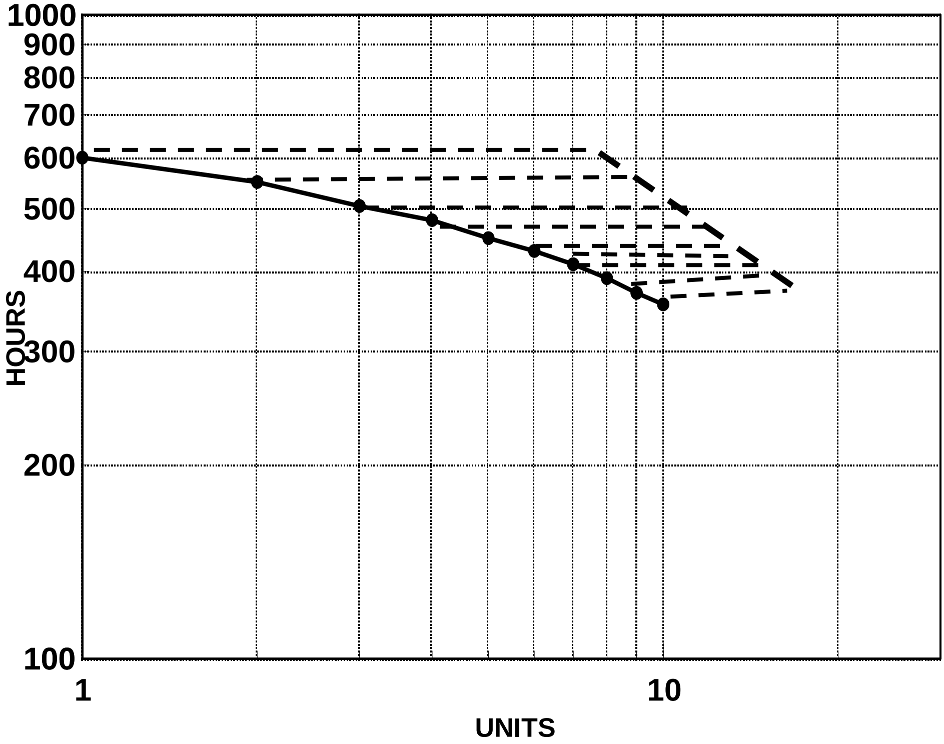
c. The retained prior improvement curve option of EZ-Quant will compute a least squares estimate of the retained improvement in terms of the equivalent number of units. The procedure repositions the curve to reflect the retained improvement and to project cost or hours for user-specified lots.

F-504 Interruptions in Production

a. A significant break in the production of an item may cause a "loss of experience (or learning)." The slope and vertical position of the improvement curve will not change, but the first unit produced after the disruption will regress to an earlier position on the improvement curve and the pattern established in the past will be repeated. A similar pattern may result when normal production is disrupted over an extended period because of a shortage of materials, unacceptable reject rate, or other factors.

b. The effect of a disruption is illustrated graphically in Figure F-5-5. In this case, an interruption between the production of units 7 and 8 caused a loss of experience of about 3.5 units. Moving the points for units 8 through 15 by 3.5 units to the left brings them into alignment with the points for units 1 through 7.

Figure F-5-4
Effect of Retained Learning

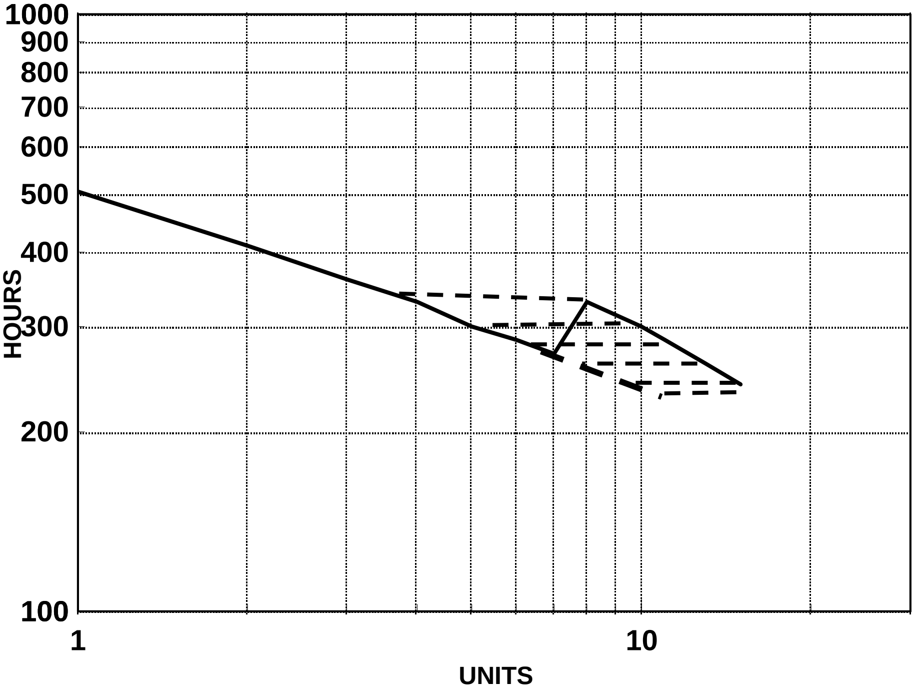


c. The break in production improvement curve option of EZ-Quant provides a least squares estimate of the improvement lost (expressed in terms of units) due to a break in production. In the process, EZ-Quant fits an improvement curve to the data, some of which is repositioned to account for the lost improvement. Data must be available for units before and after the break. The procedure also computes projected cost or hours for user-specified lots.

d. Where historical data are available only before the disruption occurred and projections of costs for units to be produced subsequent to the disruption are required, the following procedures should be considered individually or in combination to form the basis of an audit opinion.

(1) Review the contractor's methodology for an understanding of the logic employed and evaluate the computation in light of that logic. Answer such questions as: Is the contractor's reasoning consistent with the situation? Do the mathematical computations follow logically from the contractor's reasoning? Is the computed loss of learning a realistic value?

Figure F-5-5
Effect of Production Break



(2) Determine whether the contractor had breaks with production of similar items for which historical data are available both before and after the disruption. If such data are available, perform an analysis of that data with the EZ-Quant break in production improvement curve option and compare the results with the contractor's proposal considering the respective time intervals for each disruption.

(3) Request a technical evaluation of the extent of lost improvement.

e. Much of the improvement in unit costs is generally the result of better product design, tooling, work methods, and work layout. If these are properly documented, learning will not be totally lost, regardless of the length of the interruption or the turnover in personnel. Accordingly, a 100% loss of improvement (or learning) can rarely, if ever, be anticipated.

F-505 Variations in the Rate of Production

a. The rate of production can have a significant effect on unit direct labor-hour requirements if labor hours for part of the production process are fixed or semi-fixed. For example, it may be necessary for some direct workers to tend certain machines or production line stations or to perform duties related to production scheduling control, or supervision regardless of production levels.

b. Unlike engineering changes, retained learning or interruptions in production, the existence of fixed and semi-fixed labor produces no typical pattern to an improvement curve graph. Familiarity with the production process is necessary to identify situations of this sort.

c. When it is suspected that the production process may include fixed or semi-fixed labor while production rates have varied significantly, an analysis should be performed with the fixed level of effort (per time period) improvement curve option of EZ-Quant. This procedure provides a least squares estimate of the fixed portion of the observed hours and the improvement curve applicable to the variable hours. It also provides projections of future costs or hours for user-specified lots and production periods.

F-506 Other Variations in the Rate of Improvement and Cost Level

a. The improvement rate may not always appear uniform during a production run, particularly if the run occurs over a long period of time. Long-term production runs sometimes display a series of plateaus where the improvement curve is flatter than the long-term trend. These are the result of "bunching" of the implementation of improvement-inducing measures such as tool changes and production reorganization as opposed to a more gradual and continuous implementation of such measures. The long-term improvement rate is still a good reflection of the general trend of cost reduction that can be expected for future lots.

b. Contractors sometimes experience a rise in unit costs during the last few lots of a production run; the improvement curve reflects this by an upward or positive swing. There may be many reasons for a potential loss in efficiency including reduced rate of production, loss of more experienced workers, continued use of worn tools and equipment, part shortages, and worker concern for job security. Although costs might tend to go up for those reasons, there are other reasons why unit costs could actually go down (e.g., excess purchased parts and production inventory accumulated throughout the life of the program).

c. Cost and productivity improvements don't just happen---they are managed. Continuous management attention is required to ensure that costs are properly controlled. A constant rate of improvement may generally be assumed, unless the contractor can specifically document support for any proposed deviation.

F-600 Section 6 --- Application of Improvement Curve Techniques

F-601 Introduction

This section contains guidance in the use of improvement curve techniques in contract auditing.

F-602 Use with Other Analysis Methods

The improvement curve, like other statistical analysis methods, should not be considered as a complete or absolute procedure; rather, it is only an additional analytical tool useful for analyzing and forecasting cost trends when the reasonableness of the historical costs has been established by other means. While historical trends can be determined and measured with fair certainty, no future trend can be predicted with certainty. A number of variables, some of which have been discussed, can affect the forecast.

F-603 Preliminary Evaluations

In evaluating cost estimates based on an improvement curve, it is necessary to understand the reasoning behind the cost proposal and the methods of using the curve. If the contractor has constructed a curve following certain assumptions and the auditor interprets the curve from another viewpoint, his conclusions may be entirely erroneous. This does not mean that one theory or method of construction is better than another, or that the auditor should insist on only one method of construction. The basic question is this: Which approach best fits the data, especially during the early states of production? The data used to construct an improvement curve must be homogeneous; and the auditor should determine whether or not this is true. For example, if the relationship of manufacturing to subcontract work varies significantly between different production lots, an improvement curve constructed by plotting data from these lots may be inaccurate. Further, the projection of a curve assumes that the same conditions that existed in the past will be perpetuated in the future; and it is the auditor's responsibility, in evaluating a forecast, to determine whether this assumption is valid.

F-604 Use of Improvement Curves for Production Planning and Control Purposes

a. When the improvement curve is used as a guide for planning and controlling production, it is possible that the production operations may be so planned and controlled as to assure that the decline in costs as production continues will follow a preselected improvement curve pattern. This can occur when the improvement curve technique is used to establish the man-hour, space, and similar production requirements and to control their utilization for each successive production lot. Under proper conditions this method of production planning and control can result in a highly efficient operation but it also lends itself to the development and perpetuation of an inefficient operation. While the determination of the efficiency of a particular operation requires the services of a specialist who is expert in that particular field of production, there is much that the auditor can do by observing and reporting specific areas of inefficiency. These observable inefficiencies may be reflected in many ways: poor space and equipment utilization, a rate of work which is abnormally slow or clearly geared to low work requirements, production lots that are not of optimum size for efficient production, stretched out production schedules, the use of rates or norms established for discontinued production methods, workers having to wait for each other or getting in each other's way, a material flow system or shop organization that does not permit workers to work effectively, and use of old initial cost and curve slope data for control of new production situations without giving adequate consideration to past and expected changes in the product and the manufacturing methods.

b. Whether or not improvement curves are used by a contractor in controlling production, they can be used by the auditor in evaluating the efficiency of a production process. If the contractor's data shows a very low rate of improvement or other similar programs have shown higher rates of improvement, these conditions may indicate inadequate attention to improving the work methods, production line layout, and equipment and tooling used in the production process. On the other hand, an abnormally high rate of improvement for the initial units of production may be indicative of inadequate planning.

c. In making these evaluations, the auditor must be careful not to act as an expert in a field in which he does not have technical proficiency. However, he or she should be, and is expected to be an expert observer. He or she can and should be cognizant of any readily apparent material inefficiencies and weaknesses in the planning and control systems. The auditor should report facts as observed and the extent to which the improvement curve is used in production control. To the extent that a cost determination is possible, the auditor should evaluate the effect of inefficient practices on the costs. As a minimum, he or she should report his or her observations to cognizant procurement officials for further investigation and corrective action.